

Physics 11

Course Notes

Physics 11 – Course Outline

Teacher : Mr. Gaunce

Textbook : McGraw-Hill Ryerson Physics (\$120)

The textbook will be issued upon your request. It is not mandatory for you to take the textbook. If you take good notes, complete all required assignments and pay attention you will have all the information that you require to be successful in the course. The textbook may be helpful if you are having difficulty with the course or desire extra practice questions.

Course Description:

Topics covered in this course are the following:

1. Kinematics (study of motion)
2. Dynamics (study of factors that affect motion)
3. Work and Energy
4. Waves (Sound and Light)

Evaluation:

Tests/Quizzes : 40%

Labs/Assignments : 15%

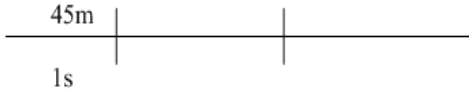
Midterm : 15%

Final Exam : 30% (Includes material covered on midterm)

Note : There will be regular assignments which will be due on the date specified. All assignments are posted on my website. It is the expectation that students print their own assignments. Late assignments will result in late marks. Lab reports have the same expectations. You will usually have 6-7 days notice for assignments unless a shorter time is agreed upon by the class as a whole. Labs will usually have 7 days for completion. Tests will not be given if the associated assignment has not been returned (assuming the assignment was submitted by the due date)

Extra help : I am available for extra help most lunch hours. This is not an overly difficult course if you keep on top of the information as you receive it. If you miss time catch up quickly as many of the concepts are based on the concept before it.

Convert 45m/s to km/h



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Converting Units, Algebra Skills and The Quadratic Formula

Pico	1×10^{12} pm	1m
Nano	1×10^9 nm	1m
Micro	1×10^6 μ m	1m
Milli	1×10^3 mm	1m
Centi	1×10^2 cm	1m
Deci	1×10^1 dm	1m
Base	1×10^0 m	1m
Kilo	1×10^{-3} km	1m
Mega	1×10^{-6} Mm	1m
Giga	1×10^{-9} Gm	1m
Tera	1×10^{-12} Tm	1m

$$2.35 \times 10^5 \text{ mg}$$

$$4.89 \times 10^{-12} \text{ GW}$$

Basic Knowledge/Background/Required Math Skills

Significant Digits

You will need to understand S.D. (significant digits) in order to understand Chemistry and Physics and to understand where certain answers come from in the answer sections in textbooks or worksheets.

Rules: Zeroes in front of number don't count

Example 1: 0.00452

0.0007506

Rules : Zeroes in whole numbers count (sometimes)

- if they are in the middle they count
- if the zero is at the end it depends on the numbers

Example 2: 53007

520

Rules for working with Significant Digits

Adding/Subtracting

Round off your final answer to have the same number of decimal places as the least precise measurement.

Example 3 : $26.442 + 10.21 + 2.1 =$

Multiplying/Dividing

Round off your answer to have the same number of significant digits as the measurement with the least number of significant digits

Example 5: $3.22 \times 2.1 =$

Example 6: $4255 \times 32.3 =$

Scientific Notation

Correct form for scientific notation is a whole number followed by a decimal and then the other digits. The end is 10 to the exponent

Example: $45,672 \times 10^6$ is incorrect

Example: 4.5672×10^7 is the correct form

To change to proper scientific notation

- use your calculator OR

- if you move the decimal place to the right you subtract the number of places from the exponent

- if you move the decimal to the left you add the number of places to the exponent

Example 7 : Write the following numbers in proper scientific notation

a) $32887.98 =$

b) $0.000034 =$

c) $876.92 \times 10^{-12} =$

d) $722.882 \times 10^{14} =$

Converting Units - The Factor Label Method

- used for changing units (which we will be doing)
- we use a ratio to eliminate units so that we can switch to the units we want

Base units have no prefix and can be **m, s, g, N, W**, etc (or just about anything as long as it doesn't have a letter in front of it)

Table of Prefixes
Each of the following are
equal to 1 base unit

Pico	1×10^{12} pm	1m
Nano	1×10^9 nm	1m
Micro	1×10^6 μ m	1m
Milli	1×10^3 mm	1m
Centi	1×10^2 cm	1m
Deci	1×10^1 dm	1m
Base	1×10^0 m	1m
Kilo	1×10^{-3} km	1m
Mega	1×10^{-6} Mm	1m
Giga	1×10^{-9} Gm	1m
Tera	1×10^{-12} Tm	1m

Note: The table above is for convert to the base unit of meters but will work for any base unit. All that changes is the base unit, so we are really only concerned with the prefixes - milli, centi, kilo, mega, etc.

Pico	1×10^{12} pg	1g
Nano	1×10^9 ng	1g
Micro	1×10^6 μ g	1g
Milli	1×10^3 mg	1g
Centi	1×10^2 cg	1g
Deci	1×10^1 dg	1g
Base	1×10^0 g	1g
Kilo	1×10^{-3} kg	1g
Mega	1×10^{-6} Mg	1g
Giga	1×10^{-9} Gg	1g
Tera	1×10^{-12} Tg	1g

Factor Label method

Procedure

1. Make a grid like below
2. Write the number and the units we want to get rid of on the top left box of a grid
3. Find the appropriate conversion using the provided tables.
4. The units we want to get rid of go in a box that is diagonal to the first units based on the ratio from the table below

Example *Have* *Want*
 Convert 421 m to cm

Step 1



Step 2



Step 3

See previous page

Step 4

$$\begin{array}{c}
 \nearrow 10^0 \\
 \hline
 421\text{m} \quad | \quad 1 \times 10^2 \text{ cm} \\
 \quad \quad \quad | \quad \quad \quad \text{m} \\
 \text{Diagonal (See ratio)} \quad 421 \times 10^2
 \end{array}
 = 421 \times 1 \times 10^2$$

**Always set up your grid so that the units are diagonal to each other. This makes the units cancel out. Also, the method we are using makes it so that the denominator is equal to 1. If you find your ratios with another method then you won't always have this happen.

*Note - If the exponent is 3 or smaller it is just as easy to convert to 1000,100, etc, which ever is the correct conversion.

Examples

Converting units using Factor Label Method

1. Convert 145cm to m

$$\frac{145\text{cm}}{\quad} \quad | \quad \text{_____}$$

2. Convert 12700mg to g

$$\frac{12700\text{mg}}{\quad} \quad | \quad \text{_____}$$

3. Convert 53.8 s to μs

$$\frac{53.8 \text{ s}}{\quad} \quad | \quad \text{_____}$$

4. Convert 239.1 Tm to m

$$\frac{239.1 \text{ Tm}}{\quad} \quad | \quad \text{_____}$$

5. Convert $1.4 \times 10^5 \text{ MW}$ to W

$$\frac{1.4 \times 10^5 \text{ MW}}{\quad} \quad | \quad \text{_____}$$

Converting from Prefix Units to Prefix Units

Examples

6. Convert 120 kg to mg

$$\begin{array}{r|l} 120\text{kg} & \\ \hline & \end{array}$$

7. Convert 1.56×10^{16} kW to GW

$$\begin{array}{r|l} 1.56 \times 10^{16} \text{ kW} & \\ \hline & \end{array}$$

8. Convert 349 mm to km

$$\begin{array}{r|l} 349\text{mm} & \\ \hline & \end{array}$$

9. Convert 7.21×10^{-6} pm to km

$$\begin{array}{r|l} 7.21 \times 10^{-6} \text{ pm} & \\ \hline & \end{array}$$

10. 54.32×10^4 ns to Ms

$$\begin{array}{r|l} 54.32 \times 10^4 \text{ ns} & \\ \hline & \end{array}$$

Converting when the units are a ratio

(Example km/h to m/s)

1. Same process as before but the second part of the unit goes on the bottom square below the first part of the unit

*The / sign or per as we say it means divide. In our method the divided values go on the bottom

Examples

11. Convert 23.9 m/min to cm/s

$$\frac{23.9\text{m}}{1\text{min}} \quad | \quad |$$

12. Convert 36 km/h to mm/s

$$\frac{36\text{km}}{1\text{hr}} \quad | \quad |$$

13. Convert 45m/s to km/h

$$\frac{45\text{m}}{1\text{s}} \quad | \quad |$$

14. Convert 108 km/h to m/s

$$\frac{108\text{km}}{1\text{hr}} \quad | \quad |$$

Metric to Imperial/English Units

The process is the same as above except you need to find the conversion factor in a text, internet, apps, etc.

15. 40mpg to km/L

*Note: the British/Canadians (sometimes) use a different conversion for L to gallons than the Americans

$$\frac{40 \text{ miles}}{1 \text{ gallon}} \quad | \quad \frac{\quad}{\quad} \quad | \quad \frac{\quad}{\quad}$$

16. You drive 400km and it takes 45liters to fill up. What is your fuel mileage in mpg?

$$\frac{\quad}{\quad} \quad | \quad \frac{\quad}{\quad} \quad | \quad \frac{\quad}{\quad}$$

In case you're wondering, if you were to buy a new car the fuel consumption is rated by L/100km.

17. Convert 8L/100km to mpg.

$$\frac{8 \text{ liters}}{100 \text{ km}} \quad | \quad \frac{\quad}{\quad} \quad | \quad \frac{\quad}{\quad}$$

Rearranging Formulae

Very Important to Success in Physics and Higher Level Math Courses

To solve for an unknown variable you want to move all of the attached variables to the other side of the equal sign based on the same principles as BEDMAS except in reverse order.

1. Get rid of any variables that are added or subtracted by doing the opposite function

(i.e. If it is added to the variable then it should be subtracted from the values on the other side of the equal sign)

2. Get rid of any variables that are multiplied or divided.

3. Get rid of any exponents or square roots

4. Get rid of any brackets

Note: The order can be changed if some simplification is done first.

Examples

1. $y = mx + b$

$$\begin{aligned}y &= 12 \\x &= 2 \\b &= 6 \\m &= ?\end{aligned}$$

2. $v = 331 + 0.6T$

$$\begin{aligned}v &= 345.5 \\T &= ?\end{aligned}$$

3. $F = \frac{mv^2}{r}$

$$\begin{aligned}F &= 200 \\r &= 15 \\v &= 4 \\m &= ?\end{aligned}$$

4. $F = \frac{mv^2}{r}$

$$\begin{aligned}F &= 120 \\r &= 12 \\m &= 4 \\v &= ?\end{aligned}$$

$$5. F = \frac{mv^2}{r}$$

$$F = 400$$

$$m = 20$$

$$v = 6$$

$$r = ?$$

$$6. x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v_0 = 16$$

$$t = 3$$

$$x = 73.5$$

$$x_0 = 12$$

$$a = ?$$

$$7. x = x_0 + v_0t + \frac{1}{2}at^2$$

$$x_0 = 0$$

$$x = 150$$

$$v_0 = 0$$

$$a = 2.5$$

$$t = ?$$

$$8. x = x_0 + v_0t + \frac{1}{2}at^2$$

$$x = 150$$

$$x_0 = 0$$

$$v_0 = 10$$

$$a = 2.5$$

$$t = ?$$

$$150 = 10t + 0.5(2.5)t^2$$

Quadratic Formula

The quadratic formula is used to determine the answer for a variable in an equation that is a quadratic function such the x in the equation below.

$$y = x^2 + 4x + 5$$

Some of you have already learned the quadratic formula in Functions and some of you haven't yet. We will use it occasionally but it will also be used in Physics 12 and Advanced Math.

The quadratic formula is shown in the box below

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

x = unknown variable (not necessarily an x in the equation)

a,b,c - are the values as they appear in the general form that is shown below

In order to use the quadratic formula you must rearrange the equation into the general form. Then you need to set the y-value to zero

$$y = ax^2 + bx + c$$

In the general form **a** is always the value in front of the variable that is squared, **b** is the value in front of the value that is not squared and **c** is the value that isn't with the unknown variable

Let's look at the question 8 from the last examples

$$8. x = x_0 + v_0 t + 1/2at^2$$

$$150 = 10t + 0.5(2.5)t^2$$

Rearrange into general form - set the left side to 0 now too.

$$0 = 1.25t^2 + 10t - 150$$

$$0 = at^2 + bt - c$$

$$x = 150$$

$$x_0 = 0$$

$$v_0 = 10$$

$$a = 2.5$$

$$t = ?$$

In this example as in many Physics questions that require the quadratic formula we use t instead of x. So when use the formula we are really finding t not x.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(1.25)(-150)}}{2(1.25)}$$

$$x = \frac{-10 \pm \sqrt{100 + 750}}{2.5}$$

$$x = \frac{-10 \pm \sqrt{850}}{2.5}$$

$$x = \frac{-10 \pm 29.15}{2.5}$$

$$x = \frac{-10 + 29.15}{2.5}$$

$$x = \frac{-10 - 29.15}{2.5}$$

$$x = 7.66$$

OR

$$x = -15.66$$

The possible answers are 7.66 or -15.66. Since time can't be negative the answer is 7.66 seconds.

Example 1. $y = y_0 + v_0t + 1/2at^2$

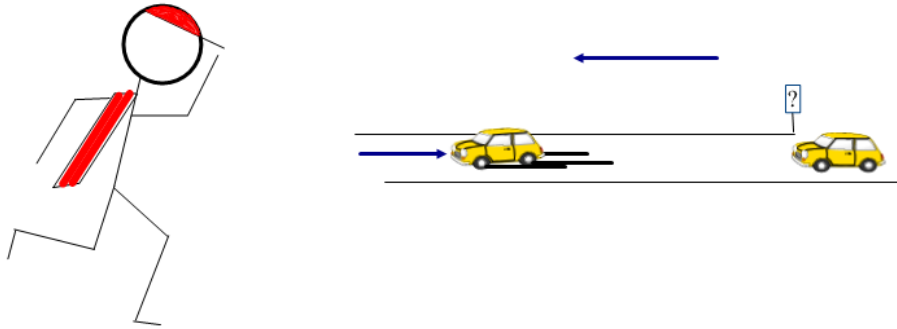
$$10 = 1 + 20t + 1/2(-9.8)t^2$$
$$0 = -4.9t^2 + 20t - 9$$

$$y = 10$$
$$y_0 = 1$$
$$v_0 = 20$$
$$a = -9.8$$

Example 2. $y = y_0 + v_0t + 1/2at^2$

$$5 = 15 + 6t + 1/2(-1.62)t^2$$
$$0 = -0.81t^2 + 6t + 10$$

$$y = 5$$
$$y_0 = 15$$
$$v_0 = 6$$
$$a = -1.62$$



Kinematics

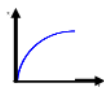
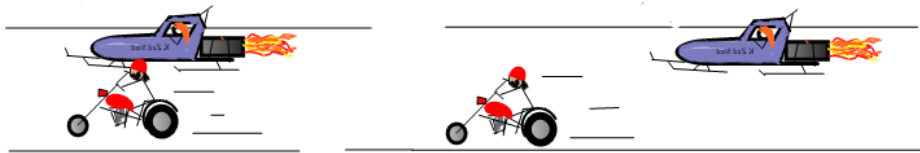
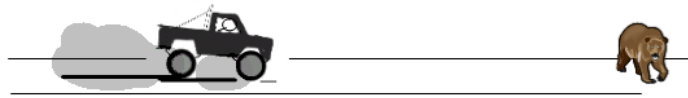


Figure 1

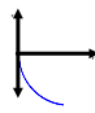


Figure 2

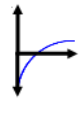


Figure 3

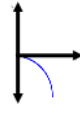


Figure 4

Physics

Unit 1 – Kinematics

Physics - the science that deals with matter, energy, motion, and force and their interactions

Kinematics - deals with the study of motion without regard for mass and force

Frame of reference - an area or position used to make measurements from. In order to discuss motion in a way that is common to everyone we must pick a point to observe from.

For example: you may be making measurements from the side of the road or in a moving car. This must be the same for everyone to get the same results. We commonly use a fixed point

Scalar - quantity that only describes a magnitude (speed, mass, time)

Vector - quantity that describes magnitude and direction (velocity, force, displacement)
- commonly denoted with a small half arrow above the variable (\vec{v})

Magnitude - simply the amount or the size of a measurement. The magnitude of your mass is the measurement of your mass.

Displacement vs. Distance

Displacement - How far an object is from its original position in a straight line. It also includes direction

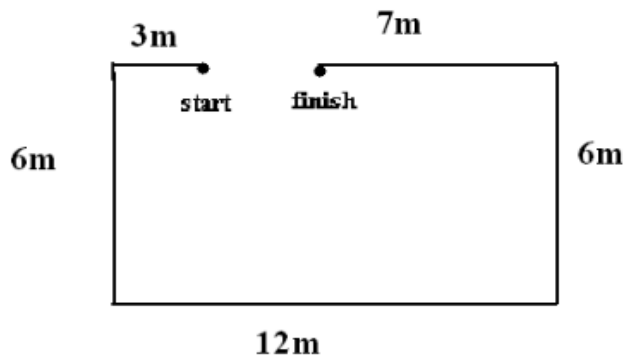
$$d = x - x_0$$

d = displacement (m)

x = final position (m)

x_0 = initial position (m)

Distance - how far an object has travelled in total. No direction as it may change several times in a journey.



Speed vs. Velocity

- Speed is a scalar
- Velocity is a vector

Both indicate how fast an object is moving but velocity indicates direction

Speed

Average Speed - total **distance** travelled divided by the total time interval of the travel

Velocity

Average velocity - is an object's **displacement** divided by the time interval over which it was travelling

$$x = x_0 + \bar{v}t$$

x = final position in the x-direction (m)

x_0 = initial position in the x - direction (m)

\bar{v} = average velocity (m/s)

t = time interval (s)

*we can use the same equation in the y-direction by simply changing any x for a y

The key to solving problems in Physics, Chemistry and Math is knowing what information we have and what we are looking for. As such we will be writing all of our known information down for each question along with a question mark for the unknown value.

Example K.1 Jimmy travelled from his farm to his grandmother's house which is 250m away. If the trip takes him 120 seconds determine his average velocity in a) m/s b) km/h

Note : If the velocity is constant then we use average velocity.

*Remember to convert between km/h and m/s

km/h to m/s

÷ 3.6

m/s to km/h

x 3.6

Remember this **only** works if you are converting between the 2 velocity measurements.

Example K.2 Johnny (Jimmy's friend) was travelling with an average velocity of 18m/s. How long would it take him to get a variety box of timbits if he lived 2.130km away from Tim Hortons? (We will make the assumption that he is going to Tim Hortons's store.)

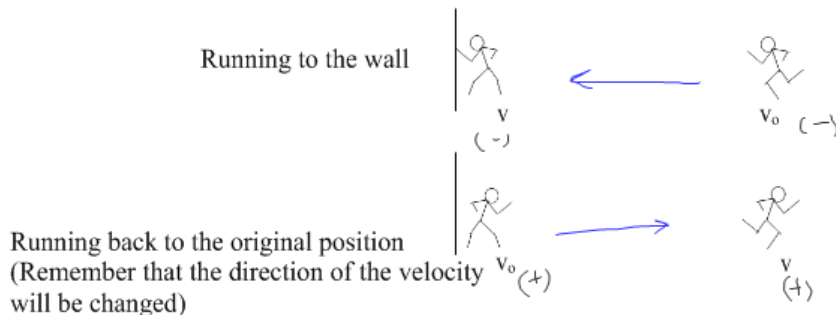
Example K.3 Roberta is driving her green 4 door Dodge Intrepid. Her average velocity is 108km/h and she travels for 30 minutes on an ol' straight country road in the fall. How far did she go in a)km, b)m?

However, in many situations the velocity will change. We should also remember that velocity can have direction therefore we will also need to account for that.

final velocity - the velocity at the end of a time interval
- denoted as v

initial velocity - the velocity at the beginning of a time interval
- denoted as v_o

Note : If there is a change in direction or acceleration during travel we may need to talk about the motion in different sections. If this is the case the final velocity of one section will become the initial velocity of the next section. An example of this would be if a person runs and touches a wall and runs back the other way. The first part of the interval is from the start to the wall. The second part is the wall back to the start. The initial and final positions change.



When solving problems watch for key words that will help determine initial and final velocities:

from rest $v_o = 0$

comes to a stop or rest $v = 0$

Average Velocity using Initial and Final Velocity

$$v = \frac{(v + v_0)}{2}$$

v = average velocity (m/s)

v = final velocity (m/s)

v_0 = initial velocity (m/s)

* This formula applies to situations where there is a constant acceleration over 1 time interval

****This DOES NOT work when you have two different average velocities over two different time intervals**

*** Care should also be taken when dealing with falling objects also as v is usually not 0.

Example K.4 Rueben was travelling at 40m/s and increased his velocity to 64m/s.

Determine his average velocity in km/h.

Example K.5 Henrietta was in her best friend's snazzy, rusted silver 2001 Honda Civic hatchback, travelling at 20m/s and increased her velocity. If her average velocity was 31.5 m/s what was her final velocity?

Keys to Solving Kinematics Problems

1. Read the question completely first
2. Identify what is being asked:
 - a) How far means you are looking for x
 - b) How fast was he going initially is v_0
 - c) How long did it take him/her is t
 - d) What is the distance is likely x and x_0 is 0 unless stated otherwise
3. Know your units
 - a) position/distance/displacement are in **m** or **km**
 - b) average velocity, initial velocity and final velocity are in **m/s**
 - c) acceleration is in **m/s²** and you can't convert it
4. Identify the type of velocity
 - a) average velocity - it will say average or constant and/or there is no mention of other velocities
 - b) initial velocity - it will say they **were** travelling at, or the person **is** and then accelerates
- started from rest
 - c) final velocity - came to a complete stop, what is he/she travelling at
5. Remember you want to have you units in **m, m/s, m/s²** for almost all of the questions you do
6. If you change km/h to m/s and end up with a really large number, you likely went the wrong way (Example: If you were travelling at 100km/h and converted the **WRONG** way you would get 360m/s which is faster than the speed of sound!)

Acceleration

Acceleration - change of velocity over a given time interval

- it can be negative or positive*
- also happens with a change of direction

$$v = v_o + at$$

a = acceleration (m/s^2)

v = final velocity (m/s)

v_o = initial velocity (m/s)

t = time interval (s)

Note : acceleration is **always** in m/s^2 , therefore the velocities must be in m/s and the distance must be in m.

Direction of Motion

- For our purposes we are going to use positives and negatives to represent direction. Right will be (+) and left will be (-). If the motion is vertical - Up is (+) and down is (-). The same convention used in math on a Cartesian plane.

Determining Direction of Distance, Displacement, Velocity and Acceleration

Example K.6 Draw arrows to represent the direction of displacement, velocity, and acceleration for the following scenarios:

- a) Abe is heading east and slows down b) Brad is heading west when he starts to accelerate
- c) Caleigh is driving south when she has to slam on the brakes d) Danika is travelling north when she begins to accelerate

Example K.7 Litisha was driving her faded red Toyota Corolla on the old highway to Penobsquis. She came upon a slow moving multi-colored farm truck, driven by Devin, and thus decided to pass. Determine her acceleration if she increased her velocity from 72km/h to a velocity of 108km/h over a time interval of 5 seconds. (Note: it was a broken line)

Example K.8 Georgia's car can decelerate with a maximum rate of 6m/s^2 . If she was driving east on the highway and came to a complete stop in 5.4 seconds a) determine her maximum original velocity in km/h, b) determine how far she travelled while she was stopping. (There was highway construction if you were wondering.)

Finding Displacement Without Final Velocity

$$x = x_o + v_o t + \frac{1}{2} a t^2$$

$$y = y_o + v_o t + \frac{1}{2} a t^2$$

x = final position (m)

x_o = initial position (m)

v_o = initial velocity (m/s)

a = acceleration (m/s^2)

t = time (s)

Example K.9 Bernie was chasing after Marshall on the school playground. Bernie was jogging at 1.1m/s initially and then accelerated at 2.3m/s^2 . a) If he runs for 3 seconds before Marshall trips how far did he travel during the acceleration? b) What is Bernie's final velocity?

Example K.10 Alvin is driving in his car when he slams on the brakes to test how accurate his G-Tech 2 meter is. If he slows his car for 200m in a time of 5 seconds determine the initial velocity if the acceleration was 2.5m/s^2 ?

Example K.11 Damon is riding his motorcycle down the highway at 90km/h when he decides to accelerate. a) What is his acceleration if he accelerates for 5 seconds over a distance of 212.5m ?
b) What is his velocity after the 5 seconds?

Vertical motion and Acceleration due to Gravity

So far we have only talked about left/right movement and the associated accelerations.

We also need to talk about motion of objects that move in free fall in the vertical (up/down) motion.

When objects freefall (stones, rocks, balls, or any falling object) they always have the same acceleration. In reality there may be other forces but for us we are dealing with only acceleration due to gravity, which is a constant near the earth's surface, of -9.8m/s^2 . If you move further away from the surface the value will decrease slightly but we won't be worried about that for now. We also use a **g** to represent **a** in the equations.

$$a = g = -9.8\text{m/s}^2 \quad g = \text{acceleration due to gravity } (-9.8\text{m/s}^2)$$

* it is negative because it causes an increase in velocity downward or in the negative direction

*Note : since objects always fall in a downward direction the displacement will be a negative value. In these situations we can use 0 to represent the final position and the starting height will be the initial position.

*Note : if an object is thrown upward gravity acts to slow it down and as such the motion is upward (+) but the acceleration (g) is still (-).

Because gravity only affects objects moving in the vertical plane we will be talking about position in terms of y . Therefore, we can change the previous equations by substituting y for x .

$$x = x_0 + v_0t + \frac{1}{2}at^2$$



$$y = y_0 + v_0t + \frac{1}{2}at^2$$

v_0 = initial velocity in the y -dir (m/s)

y = final height (m)

y_0 = initial height (m)

$a = g = -9.8\text{m/s}^2$ (when objects are falling)(This is a constant for falling objects - on Earth)



Example K.12 Some keen students decide to test this whole acceleration of gravity stuff. They go to the top of a rather short, 12m tall water tower and drop an old weathered baseball. (Since it is being dropped it had no initial velocity). a) Assuming that the value of g is correct, how long should it take for the ball to hit the ground? b) Determine the final velocity of the ball just before it hits the ground.



Example K.13 A ball is thrown upward with an initial velocity of 72km/h. How high will the ball be if it is in the air for, let $y_0 = 0\text{m}$, a) 2 seconds b) 4 seconds? c) What is the velocity at the maximum height?

Variables Quiz – Time for a check of your ability to identify the variables in the questions

Multiple Choice

1. What symbol represents final velocity?
a. v_o b. v c. \bar{v} d. none of the previous
2. Which of the following units would represent average velocity?
a. m b. km/h c. m/s^2 d. none of the previous
3. Martin walked from the school to Mrs. Dunsters and back during lunch time. Mrs. Dunsters is about 250m away. If it took him 10 seconds his average velocity would be?
a. 0m/s b. 50m/s c. 25m/min d. 25m/s

Use the following question to answer multiple choice questions 4,5,6 and 7 below

Raymond is driving his car at 54km/h east when sees the stop light. If it takes him 5 seconds to come to a stop, what is his acceleration?

4. What variable are we looking for?
a. v_o b. \bar{v} c. v d. a
5. What is the final velocity?
a. 270m b. 54km/h east c. 15m/s east d. 0km/h
6. Which of the following is likely his initial position?
a. 0m b. 30m/s c. 150m d. 9000m
7. Which of the equations below would you use to solve for missing variable? (See list below)
a. Equation 1 b. Equation 2 c. Equation 3 d. Equation 4

Use the following question to answer questions 8,9,10 below.

Estelle drives her car up a highway on-ramp. She gets her car up to a velocity of 108km/h north when she hits the highway. Her average velocity on the ramp was 72km/h north. What was her velocity when she started on the ramp

8. What variable is represented by 72km/h represent in the question above?
a. v_o b. v c. \bar{v} d. a
9. What variable are we looking for in the question above?
a. v_o b. v c. \bar{v} d. x
10. Which of the following would most likely be the answer?
a. 180km/h north b. 40m/s north c. 10m/s north d. 144km/h north

Acceleration Situations Without Time

Occasionally we don't have the time but we have the other variables. If you combine the other formulae we can get the following formula. This formula removes the time variable that is often the largest source of error in these types of problems

$$v^2 = v_o^2 + 2a(x - x_o)$$

v = final velocity (m/s)

v_o = initial velocity (m/s)

a = acceleration (m/s²)

x = final position (m)

x_o = initial position (m)

We can also use the following variation for object travelling in the y-direction:

$$v^2 = v_o^2 + 2a(y - y_o)$$

a = g

Example K.14 Tommy is cruising down the road when he spots a police car up ahead. Over a distance of 200m, Tommy manages to slow his car to the posted speed limit of 90km/h. If he slows down at a rate of 3.5m/s², determine his velocity when he spotted the police car.

Example K.15 A mechanic accidentally leaves a 3/4 inch ratchet in the open doorway of a helicopter. The helicopter is moving upward at 5m/s and is at a height of 200m when the vibration causes the ratchet to fall out. a) What is the velocity of the ratchet after 3 seconds? b) What is the height of the helicopter after the 3 seconds? (Note: the helicopter moves upward at a constant velocity) c) What is the height of the ratchet after the 3 seconds? d) How far apart are the ratchet and the helicopter at this time?



Example K.16 Adam throws a ball straight upward in the air at 10m/s from a height of 1.5m. Determine the maximum height of the ball.

Example K.17 Isabelle is doing some science experiments. She wants to know how high she has to be in order for her test projectile to hit the ground below at a velocity of 90km/h.

- a) Determine the required height as well as the time in the air.(Assume she dropped it)
- b) Determine the velocity at the ground if she launched the projectile, from the height found in part a), with an initial velocity of 10m/s downward. Is it different for upward velocity than it would be for downward?

If you are still struggling at this point it is probably time to look at the supplementary notes on the website.

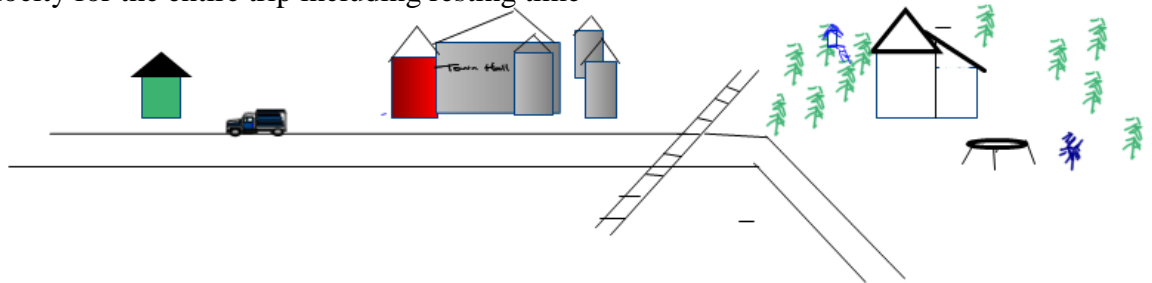
Finding Average Velocity over Different Time Intervals

As you hopefully noticed there are two formulas that can be used to find average velocity. The first equation uses position and time, the other uses initial and final velocity (Check them out on your formula page – you do have a formula page right?)

When you read the next example you will note you are given the time for each interval. Treat each time interval as a separate question. Each final position becomes the initial position for the next time interval.

Example K.18 Kirk decides to go jogging. He leaves his house which is 1.5 km west of town at 6pm. He runs to his friend James' house which is 2km east of town and arrives at 6:45pm. He stays for an hour and then jogs back toward town. He stops in town at 8:15pm and talks to Lars. At 8:30pm he starts jogging back home but gets picked up 0.5km from his house by Robert, at 8:45pm, who is mad that he is jogging this late and it is almost dark. Use the sketch to show Kirk's progress and determine the following:

- Average velocity for the trip to James' house
- Average velocity from James' house back to town
- Average velocity from town until Robert picks him up
- Average velocity for the entire trip including resting time



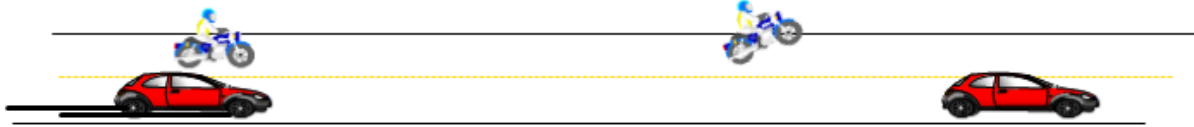
Example K.19 Edwin is doing a run-swim competition. His average velocity during the run section is 6m/s west and he runs for 1 hour. In the swim section he swims for a half of an hour and he averages 3m/s west. a) Determine how far he travels in total. b) Determine his average velocity for the entire competition.

Alright, it is time to apply a bit of what you know and expand a little. Read the questions and think about what is going on before you start (you should always do this actually) Label the diagrams with information too.

Example K.20 Marvin was travelling in his jacked up, primer black 1978 GMC Sierra 4 x 4 stepside. (It is still primer because he spent his money on tires and wheels). If he is travelling at 120km/h and slams his brakes on to avoid hitting a 10 point buck, which is 140m away, determine whether he gets to a complete stop if the rate of deceleration is 3.78m/s^2 . (*Note you are looking for the final velocity OR assume final velocity is zero and find the distance – one piece of information is for reference not calculation)



Example K.21 Carl has challenged Gretta to an impromptu challenge of speed. Both will start at 50km/h. Determine how far apart will they be when Gretta reaches a velocity of 140km/h. Carl's acceleration is 4.5m/s^2 . Gretta's acceleration is 3.5m/s^2 .



Example K.22 Howard and Jennifer are out for a day of mud slinging fun when Howard's Buick powered Toyota truck breaks. To get home they tie a rope to Jennifer's Big Block Ford powered Jeep Cherokee and head west, towards home. They are travelling at constant of 30m/s when, unbeknownst to Jennifer the rope breaks. Howard slows down at a rate of 2.2m/s^2 while Jennifer continues on.

- How far does Howard travel before he stops?
- How long does it take to stop?
- How far does Jennifer go during the time it takes Howard to stop?
- How far apart are they going to be by the time he stops?



Instantaneous Velocity

Instantaneous velocity - the velocity at any instant in time. The speedometer in your car tells you instantaneous _____. If you have a compass on your mirror it would tell you the direction to allow you to find the instantaneous _____.

Dot Diagrams to Represent Motion

We can use dots to represent the position of an object over a certain time interval. You can tell by the space between the dots whether the object is travelling at a constant velocity, speeding up or slowing down. Also if the dot diagrams are drawn to scale you could determine velocities, acceleration, etc.

The following are some examples. What is happening in each? (Imagine that you are walking with a stick in sand and poking the ground at constant time intervals – like once every second for example)

1. • • • • • • •

2. •• • • • • •

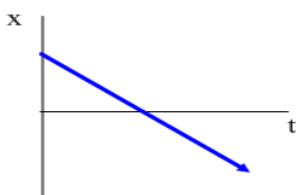
3. • • • • • • •

Interpreting Graphs

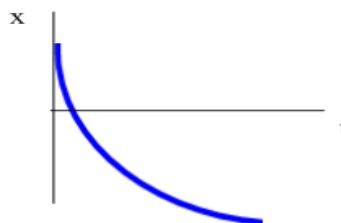
Understanding how to interpret the information on a graph is skill necessary for creating other graphs in Physics. Like graphs in Math, graphs in Physics use the x and y axis with x as your independent and y as your dependent variables (what happens to y depends on x but x doesn't depend on y). For this unit we are concerned with position (for displacement), velocity and acceleration. Position, velocity and acceleration are all dependent on time, therefore they are the dependent variables and are on the y-axis and time is the independent variable and is on the x-axis.

Example K.23 Answer the following questions based on the following diagrams – take note of the variable of the dependent axis.

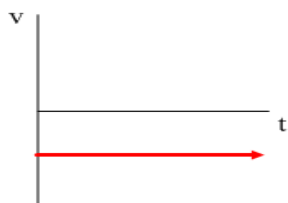
- Where does the graph start?
- What happens as time increases?
- Does the dependent variable change by a constant amount or does it vary? Does it change at all?
- What does each graph tell you?



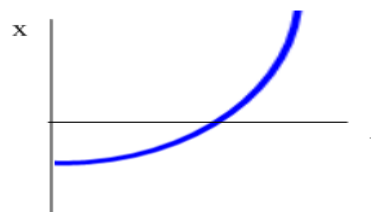
Graph 1



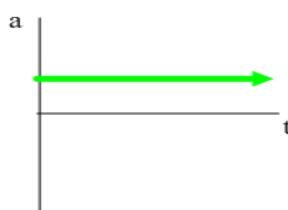
Graph 4



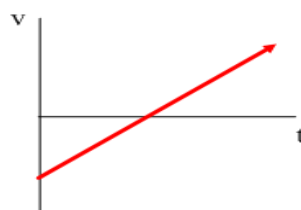
Graph 2



Graph 5



Graph 3



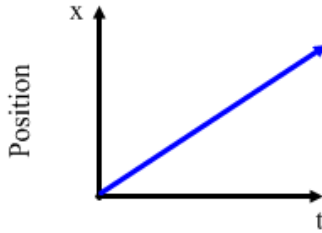
Solution: Graph 1

- The graph starts in the positive (east or north)
- The position of the object moves towards the west (or south)
- The dependent variable changes by a constant amount.
- The object starts in the east (or north) and moves west (or south) at a constant rate.

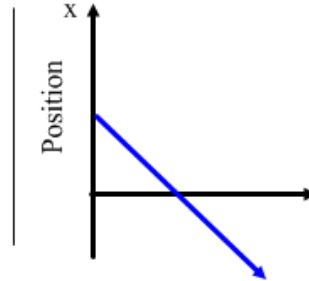
Graphing of Position vs. Time and Velocity vs. Time

1. Constant Velocity - straight lines on a Position-time graph

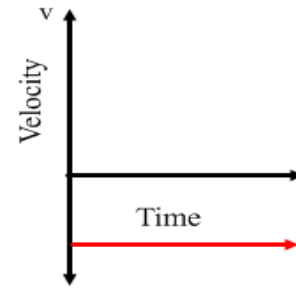
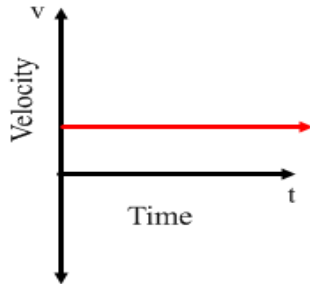
Moving away from starting point



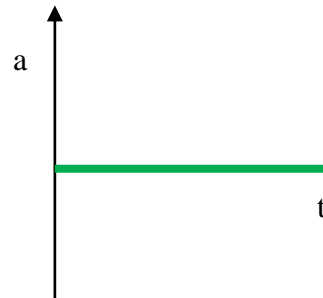
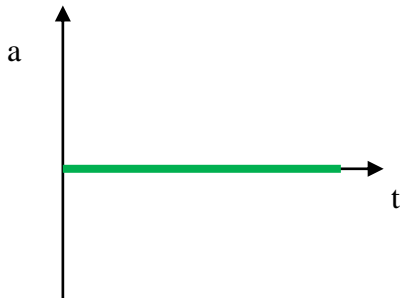
Moving away from starting point



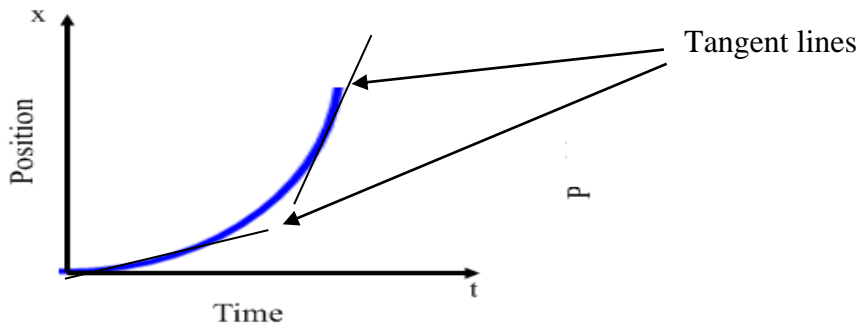
We can create a graph of velocity vs. time by taking the slope of the position vs. time graph. (The derivative of the position equation if you have the equation)



We can find the acceleration of each object represented in the graphs by taking the slope of the velocity-time graph. If you look at the slope in either case it is 0 (There is no steepness to either graph). Logically this makes sense since an object that moves at a constant rate has no change in velocity and therefore no acceleration



2. Positive Acceleration - Curved lines on a Position-time Graph

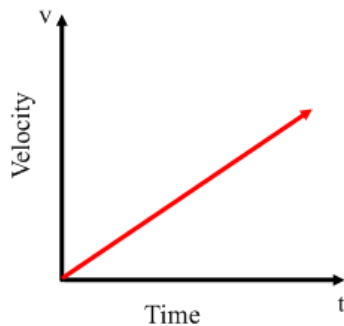


To create the velocity-time graph you can draw 2 tangent lines on the curve and determine whether the slope of the tangent lines is + or - and whether it is getting steeper or flatter.

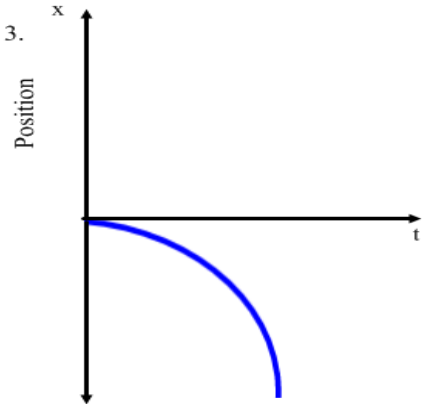
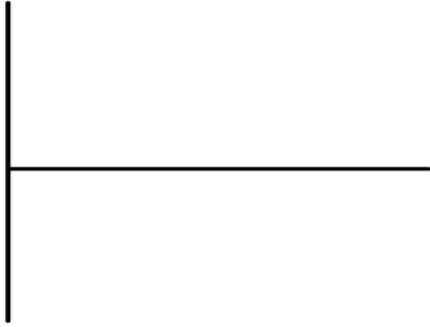
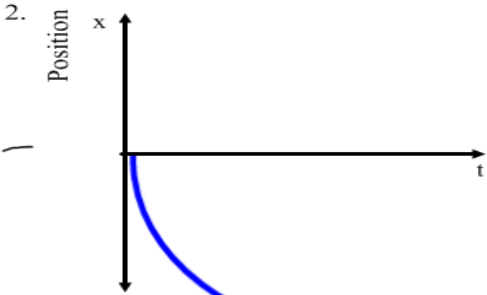
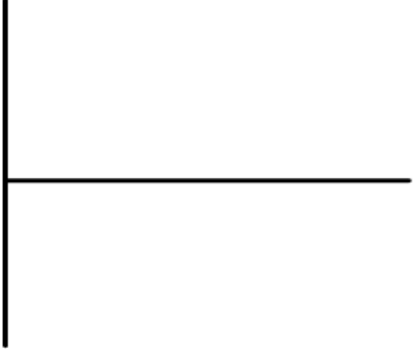
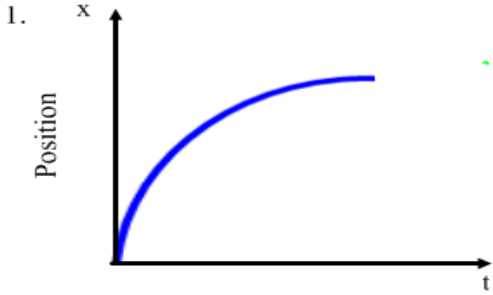
A **tangent**, by the way, is a line that touches a curve or circle at just one point. Note that the tangent lines above touch at just one point.

Each tangent represents the velocity at that point in time on a position-time graph.

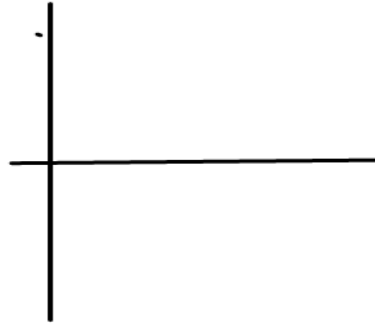
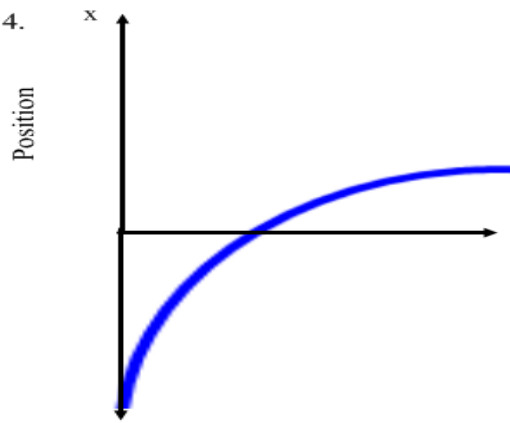
- Positive slopes represent positive velocities and are therefore on the positive side of the y-axis.
- Negative slopes are on the negative side of the y-axis.
- If the slope of the tangent lines are getting steeper the graph moves away from 0.
- If the tangent lines are getting flatter the graph goes toward 0.
- The starting point of the position-time graph does not affect the velocity time graph.



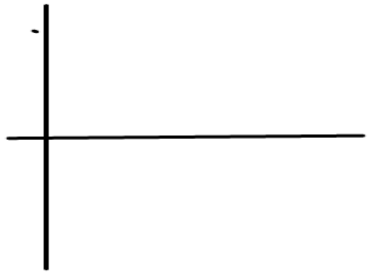
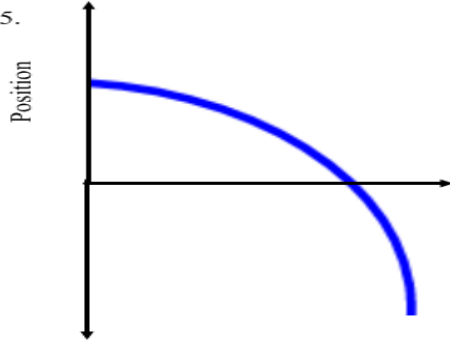
Example K.24 Create V-t graphs for the following P-t graphs

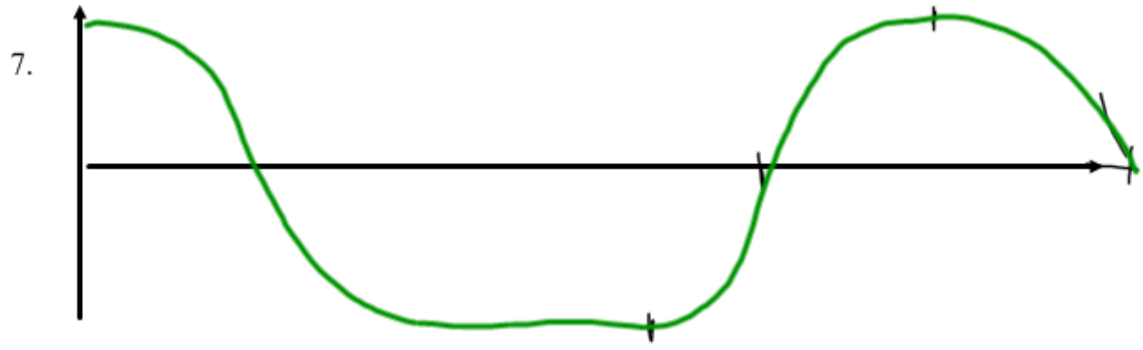
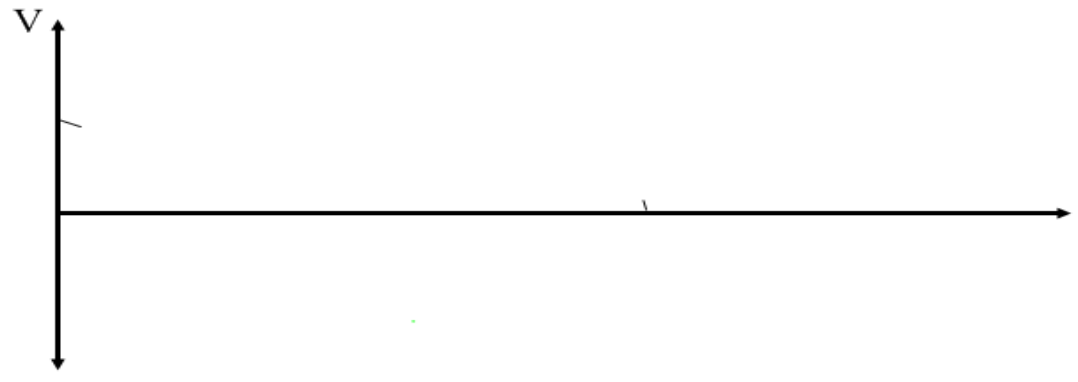
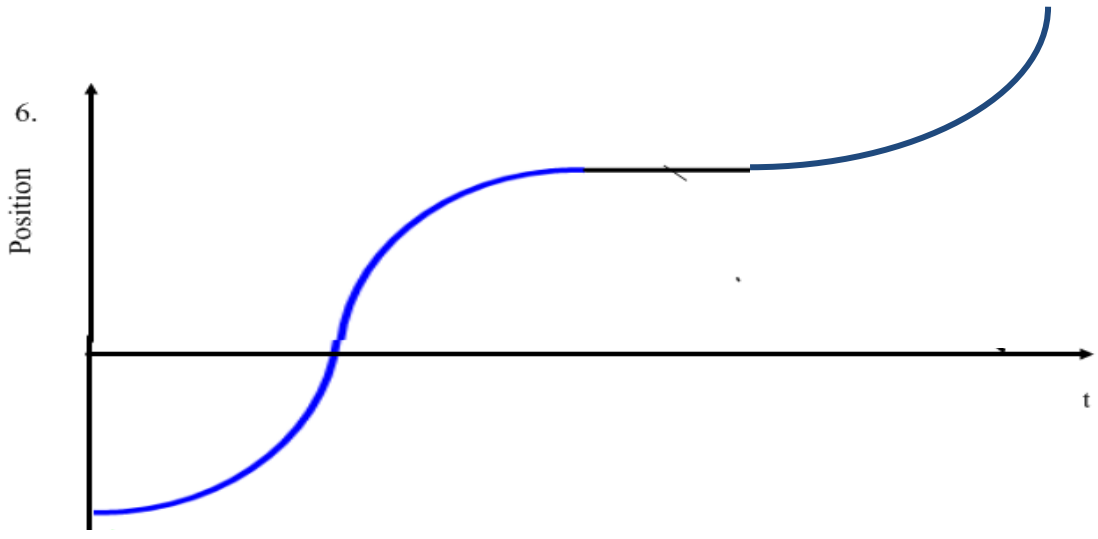


4.



5.





Relative Velocity

Sometimes our frame of reference is a moving object. If we want to determine the velocity of one object relative to another it is simply the velocity of the object in question minus the velocity of the person observing.

$$V_{\text{relative}} = V_{\text{object}} - V_{\text{observer}}$$

Example K.25: A car is travelling at 20m/s east. A truck is travelling at 30m/s east.

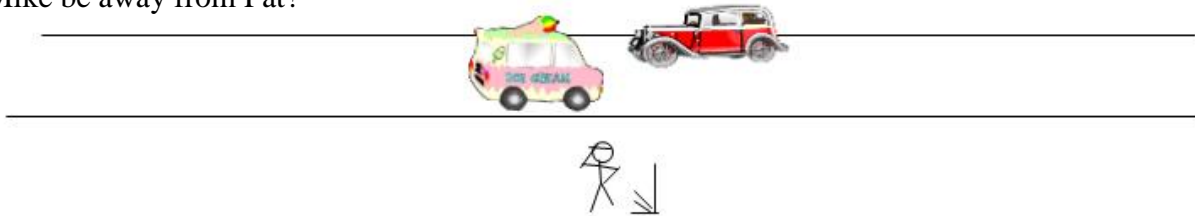
- What is the velocity of the truck relative to the car?
- What is the velocity of the car relative to the truck?

Example K.26 Markus is driving in his red Dodge Ram at a velocity of 120km/h. Bart is travelling west at 95km/h. Determine Bart's velocity relative to Markus if a) Markus is headed east b) Markus is headed west

Example K.27 Brendan A. is travelling in Wanda's black Civic at 120km/h west in a 50km/h zone. (He's cool!) He meets Chad who is going the other direction. If Chad's velocity relative to Brendan is 65m/s east, determine Chad's velocity.

Example K.28 Murray and Shane are travelling on the same road (the Penobsquis road). Murray appears to have a velocity of 20m/s east relative to Shane (the observer). If Shane is travelling at 84km/h east, determine Murray's actual velocity.

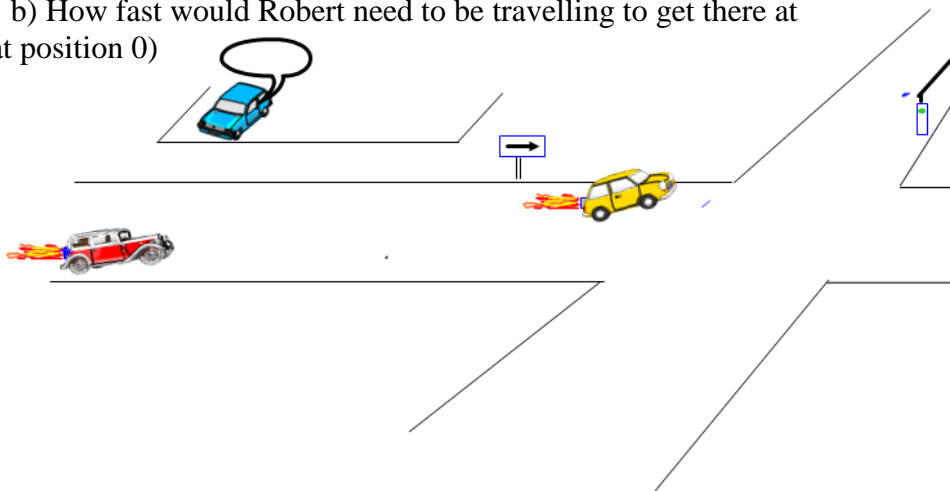
Example K.29 Mike is travelling west at 108km/h and he sees Tony standing beside a stairmaster trying to figure out how to operate it. At the same time Pat meets him in his multi colored ice cream truck. a) If Pat's velocity relative to Mike is 200km/h east determine how fast Pat was travelling. b) After 5 minutes how far would Mike be away from Tony? c) How far will Mike be away from Pat?



The following questions are slightly more difficult in that they apply multiple concepts and/or formulae.

Example K.30 Annie is sitting in her car 'talking' about everything and anything.

True story. At the same time Rob drives by her location at a constant velocity of 108km/h. Up the road at a distance of 200m Joe is sitting at a stoplight in his rocket car. When Rob passes Annie the light turns green and Joe stomps the loud pedal (the gas) and accelerates his hot rod at 5m/s^2 . a) Who would get to the 600m mark first (relative to Rob)? (Joe has a head start but he doesn't use the juice). b) How fast would Robert need to be travelling to get there at the same time as Joe? (Robert is at position 0)

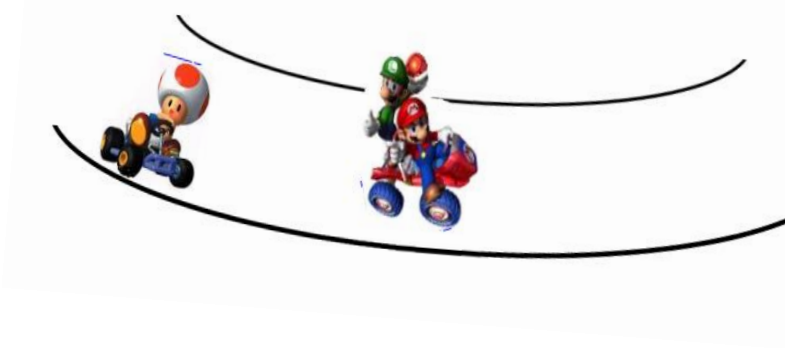


Example K.31 Mertle and Wanda are driving in opposite directions. Mertle is travelling west and Wanda is travelling east. At the moment they meet they each start accelerating. Mertle can accelerate at 3.5m/s^2 and Wanda can get her car to increase at 4.1m/s^2 . a) If they each reach a speed of 130km/h after 5 seconds determine their original velocities b) How far are they apart after the 5 seconds?

Example K.32 Mario and Luigi have made some mods to their mario kart and decided to have a competition of speed with Toad at Peach Beach. Mario and Luigi's kart has acceleration capabilities of 3.5m/s^2 , Toad's kart has an acceleration of 4.0m/s^2 . If Mario and Luigi start at 54km/h and Toad's starts at 60km/h

a) Determine how far apart they are after 4 seconds

b) After the first 4 seconds Mario hits the mushroom and accelerates for the next 3 seconds. How fast must he accelerate in order to be at the same position as Toad after the 7 seconds?



Example M.33 Geena, Cale, Taylor

Geena is driving um, her dad's oversized, gas guzzling, Earth destroying, fossil fuel eating truck. She starts at a position of 20m. Cale is driving a Smart (?) car (they don't look that smart on the highway in the wind). He starts at a position of 200m. Taylor is driving an apple green, 68 VW bug with chrome exhaust and no catalytic convertor! She starts at a position of 0m.

Determine who gets to a position of 5km first based on the following specifications:

- Geena's truck - acceleration is 2.5m/s^2
 - Cale's car (if you call it that) - acceleration is 1.8m/s^2
 - Taylor's bug - acceleration is 3.1m/s^2
-
- Geena accelerates for 15 seconds, starts to worry she is going to run out of gas and then maintains a constant velocity.
 - Cale accelerates for 20 seconds and starts to worry about the safety of the car and maintains a constant velocity
 - Taylor accelerates for 12 seconds, the water temp starts to rise since the engine is in the trunk and then slows for 2 seconds at 1.0 m/s^2 and then maintains a constant velocity.

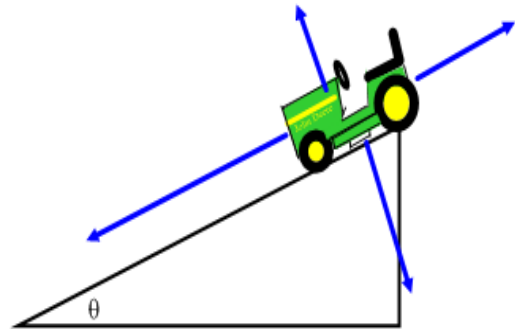
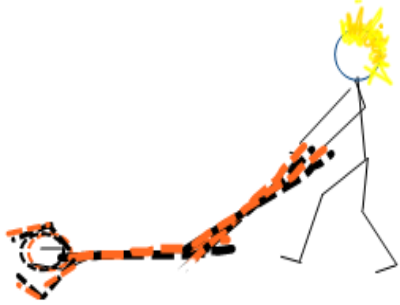


Physics 112/111 – Kinematics Practice Worksheet

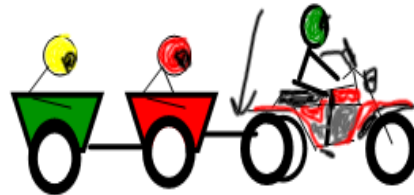
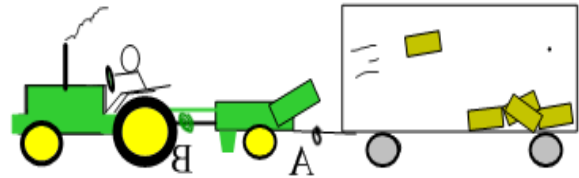
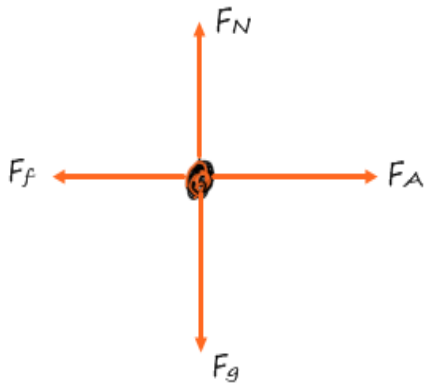
1. A motorist travels 400 km during an 8.00 hour period. What is the average velocity in (a) km/h (50km/h) (b) m/s? (13.9m/s)
2. A bullet is fired from a rifle with a velocity of 360.0 m/s. (a) What is the time required for the bullet to strike a target 1620.0 m away? (4.5s)
(b) What is the velocity of the bullet in km/h? (1296km/h)
3. An electron travels through a vacuum tube 2.00 m long in 1.60×10^{-3} sec. What is the average velocity of the electron in (a) m/s (1250m/s)
(b) cm/s? (125000cm/s)
4. In a four hour period, a hiker walked 6.00 km during the first hour and 4.80 km during the second hour. After resting for an hour, the hiker walked 3.20 km during the fourth hour. What was the hiker's average velocity during: (a) the first two hours (5.4 km/h) (b) the first three hours (3.6 km/h) (c) the entire four hour period? (3.5 km/h)
5. Light from the sun reaches the Earth in 8.30 min. The speed of light is 3.00×10^8 m/s. How many km is the Earth from the Sun? (1.49×10^8 km)
6. Show vector arrows for the displacement, velocity and acceleration if a) a car heads south and slows down. b) A car heads west and accelerates.
7. A truck starting from rest travels 200.0 m in 20.9 seconds. Find the average speed, the final speed, and the rate of acceleration. (9.57 m/s, 19.14 m/s, 0.92 m/s²)
8. A jet plane, starting from rest, is accelerated uniformly to its takeoff speed of 50.0 m/s in a time interval of 5.00 seconds. (a) What is the planes acceleration? (10m/s²) (b) What minimum length of runway is needed to accomplish this? (125m)
9. A car starts from rest and undergoes an acceleration of 3.00 m/s² for 337.5m. How long does it take to travel this distance? (15.0s)
10. A person applies the brakes to his car and has an acceleration rate of -9.00 m/s². If it takes 8.00 sec. for the car to come to a halt, (a) what was the original velocity of the car; (72m/s) (b) what is the displacement during this time? (288m)
11. A car traveling at 60.0 m/s is slowed to 20.0 m/s in 20.0 seconds. Find the distance traveled during deceleration. (800 m)
12. A train moving at a velocity of -15.0 m/s is accelerated uniformly to a velocity of -45.0 m/s over a 12.0 sec period. (a) What is its acceleration? (-2.5m/s²) (b) What is the train's average velocity during the acceleration? (-30m/s) (c) What is the train's displacement during the acceleration? (-360 m)
13. A car is accelerated from rest at a constant rate of 1.5 m/s² for a distance of 500 m. What is the velocity of the car after traveling to this position? (38.7 m/s)
14. A baseball player throws a ball vertically upward with an initial speed of 30 m/s and an initial height of 1.5m. Find the height of the ball after (a) 2 s (b) 4 s (c) 6 s. (41.9m, 43.1 m, 5.1 m)
15. A boy standing on a cliff that is 39.2 m high drops a stone. (a) Find the time taken for the stone to hit the bottom. (b) Find the final velocity as the stone hits the bottom. (2.83s, -27.7m/s)
16. Hank is driving at 20m/s when he accelerates for 4 seconds at a rate of 2.5m/s². Determine his a) final velocity b) displacement (30m/s, 100m)
17. Theo is travelling at an unknown speed when he needs to stop suddenly. His car can decelerate at 5m/s². If travels 240m while coming to a stop determine his initial velocity. Assume he is travelling east. (48.99m/s)
18. Jared is heading west at a constant velocity of 36km/h. Mia is headed east at 18km/h when she accelerates at 2m/s². How far apart will they be after 5 seconds?(100m)
19. Riley is sitting in a helicopter hovering at 400m when he drops his sandwich. a) How fast is the sandwich going after 4 seconds
b) How long does it take for the sandwich to hit the ground? (Ignore air resistance)(-39.2m/s, 9.04s)
20. Justin is travelling at 72km/h at a position of 200m. What acceleration would he need to have a final velocity of 216km/h as he passes a 1km marker? What color is his car? (2m/s²)
21. A box falls off the back of a truck and slides along the street for a distance 30.0 m before coming to rest. If friction decelerates the box at a rate of 3.00 m/s², with what velocity was the truck moving when the box fell? (13.4 m/s)
22. A motorcycle accelerates for a distance of 100m to a velocity of 144km/h. If the rate of acceleration was 3m/s² what was the initial velocity? (113.84km/h)
23. Determine the acceleration of Hank's moped if he can accelerate from 45km/h to 93.5km/h over a distance of 180m. (1.44m/s²)
24. A golfer drives a ball into the air and it takes 4.80 seconds before it hits the ground. What maximum height did the ball reach? (Hint: use the second part of the ball's trajectory) (28.22m)
25. An experimental rocket car moves along a straight track at a constant speed of 900 m/s. The car passes a group of officials, travels a distance of 270.0 m, and then explodes. If the officials hear the sound of the explosion 1.10 seconds after the car passes their position, what is the speed of sound? (337.5 m/s)

Level 1

26. Two airplanes are 10.0 km apart and are moving at constant speeds in the same direction. The second plane overtakes the first plane in 2.50 hours. (a) What is the relative speed of the first plane with respect to the second? (-4 km/h) (b) If the ground speed of the second plane is 400 km/h, what is the ground speed of the first plane? (396 km/h)
27. How long is the ball in question 14 in the air for if it lands on the ground? What is the maximum height the ball reaches? What distance does the ball travel in total? What is the displacement of the ball? (6.17s, 47.42m, 93.34m, -1.5m)
28. Just as the light turns green, a stationary car starts off with an acceleration of 3.00 m/s². At the same instant, a truck traveling in the same direction with a constant speed of 12.0 m/s passes in the next lane. (a) What minimum distance will the car have to travel in order to catch up with the truck? (b) What speed will the car be going when it catches up with the truck? (96m, 24m/s)



Forces



Dynamics - Force

Definitions

Force - in most basic terms it is either a push or a pull

Friction - is a force that acts against the motion of an object. There are two types:

1. static friction - force that opposes the start of motion
2. kinetic friction - force that acts against an object once it has started moving

Applied Force - force that drives or moves an object forward, we look at whatever is driving the object forward in the question.

Force of Gravity - the downward force on a body due to the gravitational pull of the earth (more to come later)

Normal Force - the force applied on a body by surface that it is in contact with. It always act perpendicular to the surface

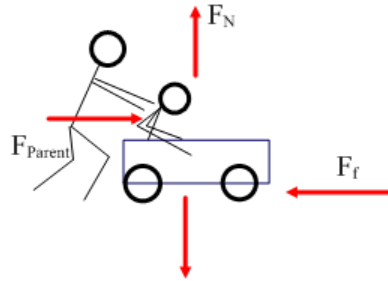
Net Force - the sum of all the forces acting on an object in one plane (horizontal or vertical).
- *All the forces in the positive direction minus all the forces in the negative direction.*

Free body Diagram (FBD) - simplified diagram used to represent all the forces acting on an object. We will represent the object with a dot.
- each force is represented with an arrow indicating the direction of the force
- any time we are dealing with a situation that has forces a FBD is required as well as an F_{net} equation.

How to create an FBD

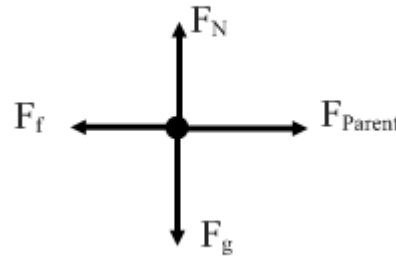
- Determine what the forces are being applied to in the question
- Draw a dot or a picture if you prefer
- Apply vector arrows for each force that is being applied
- Label each force based on the types of forces discussed above - we use a capital F with a subscript that describes the force (ex. F_f means the force of friction)
- Create an F_{net} equation for the object in question

Let's look at an example. A parent is pushing their child in a small wagon. Draw a FBD and create a F_{net} equation.



We are looking at the wagon so there is the force from the parent. We can use F_p . There is also friction against the motion of the wagon. We can use F_f . Of course there is the force applied by gravity - F_g . And the last force in this case is the normal force applied by the ground. We will use F_N .

To simplify we can replace the diagram with a simple dot and apply the same forces to create a F_{net} equation.



You should recall that F_{net} is all the forces in the positive direction minus all the forces in the negative direction. We can do F_{net} in the x-plane and F_{net} in the y-plane. (See above)

When we do a F_{net} we are usually concerned with the **plane in which the object is moving**. In this case it is the x-plane.

Since F_{parent} is a force to the right (positive) and F_f is to the left (negative) they are the forces we are concerned with. Therefore F_{net} would be:

$$F_{\text{net}} = F_{\text{positive}} - F_{\text{negative}}$$

$$F_{\text{net}} = F_{\text{parent}} - F_f$$

Example F.1 Create FBDs and F_{net} equations for the following situations:

- a) Allison is pulling sled of wood.
- b) A horse is pulling a cart
- c) A car is accelerating away from a stop sign
- d) A driver slams his brakes on to avoid hitting a platypus

Newton's 3 Laws of Motion

Newton's First Law - Inertia

An object at rest will stay at rest until an external force is applied to it.

OR

An object in motion will stay in constant motion until an extra external force is applied to it.

This means that all the forces are balanced in all directions.

$$F_{\text{net}} = 0$$

$\therefore F_+ = F_-$ This only applies for constant velocity

Newton's Second Law

The acceleration of an object is directly proportional to the net force divided by the mass.

in terms of an equation

$$F_{\text{net}} = ma$$

F_{net} = net force (N)

m = mass (kg)

a = acceleration (m/s^2)

Example F.2 Thomas is pushing a 200kg sled full of pumpkins across his yard (If you had to carry a bunch of pumpkins you would use a sled too). a) If he pushes with a constant velocity and the force of friction is 200N determine the applied force. b) After a little time he pushes with 300N (assume the frictional force is still 200N), determine the acceleration if the mass is 200kg. Draw a free body diagram for each part.

Example F.3 Carlos is pushing an old fridge box full of Nintendo wrestling games and old pet rocks. He determines, with some fancy measuring thingies, that his acceleration was 1.5m/s^2 . The mass of the box was 50kg and the force of friction was 75N . Determine the applied force. Draw the free body diagram (FBD).

Newton's Third Law

For every force there is an equal and opposite force.

This means that for every force you apply there should be an equal force applied in the opposite direction.

$$F_{b \text{ on } a} = -F_{a \text{ on } b}$$

For example: When you push on the floor to move forward the floor applies an equal force back (with friction) to allow you to move.

We will do examples of Newton's Third Law at a later time.

Weight and Mass

Weight and mass are directly related but they are not the same. Your mass is the amount of matter you have and is constant regardless of location.

Weight is the force of gravity exerted on your mass. Weight and force of gravity are the same thing. We will use them interchangeably but we will denote them as F_g

Force of gravity is equal to the mass of an object multiplied by the acceleration of gravity.

$$F_g = mg$$

F_g = force of gravity (weight) (N)

m = mass (kg)

g = acceleration of gravity (m/s^2)

* g will be 9.8m/s^2 unless otherwise stated

*Note: we have dropped the (-) because the force arrows indicate direction now.

Example F.4 By putting a bucket of water on a scale it was determined that the weight on earth was 490N. a) Determine the mass of the water and bucket. b) Determine the weight of the bucket and water on the moon ($g=1.62\text{m/s}^2$) *Assume we can get to the moon. c) What is the direction of the weight of the bucket on Earth with respect to the earth's surface (up, down, right, left?)

Friction

Frictional force - force of resistance and is a contact force
- exists only when two objects are in contact with each other.

Force of friction is dependent on the surfaces in contact and the mass of the object (not always directly). **The applied force does not affect the force of friction.

Remember that we have two types of friction: 1. static 2. kinetic (see previous notes)

Surfaces	Coefficient of Static Friction	Coefficient of kinetic friction
rubber on dry solid surfaces	1	1
rubber on dry concrete	1	0.8
rubber on wet concrete	0.7	0.5
steel on steel (unlubricated)	0.74	0.57
steel on steel (lubricated)	0.15	0.06
ice on ice	0.1	0.03
lubricated ball bearings	<0.01	<0.1
synovial joint in humans	0.01	0.003

coefficient of friction - ratio of amount of friction that exists between two different surfaces
- denoted by the symbol μ

Normal force

In addition to the coefficient of friction the force of friction is also dependent on the normal force. When an object applies a force to a surface the surface applies a force back at an angle perpendicular to the surface. This is the **normal force**. For our purposes this semester the normal force will equal the force of gravity.

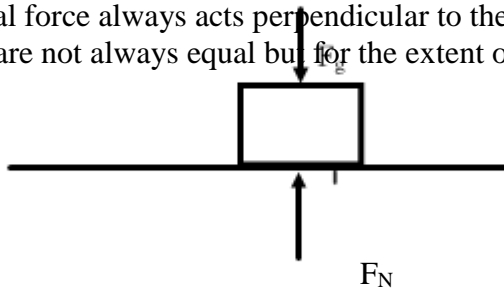
$$F_g = F_N$$

F_g = force of gravity (N)

F_N = normal force (N)

*Note: the normal force always acts perpendicular to the surface - force of gravity does not.

** These forces are not always equal but for the extent of our purposes we can assume they are equal



$$F_f = \mu F_N$$

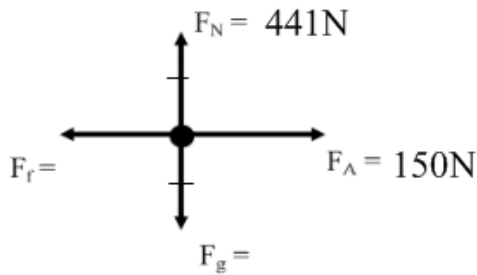
F_f = force of friction (N)

F_N = normal force (N)

μ = coefficient of friction (no units)

Example F.5 In order to keep from spinning the tires on a pickup truck, owners often put sandbags in the back of their trucks. The mass of the truck and the sandbags over the rear wheels is 2000kg. a) Determine the normal force acting on the truck tires. b) Determine the force of static friction between the truck tires (rubber) and wet concrete.

Example F.6 Determine the coefficient of friction based on the FBD provided. Acceleration is -2.5m/s^2 .



Example F.7 Kirk is pulling Morgan on a wooden sled. The mass of the sled and Morgan is 60kg . a) Determine the sled and Morgan's weight. b) If Kirk is pulling Morgan at a constant velocity with a horizontal force of 80N determine the coefficient of friction. c) What type of friction is this?

Example F.8 Rebecca wants to push a steel barrel(50kg), full of turnip patch dolls and MacGyver VHS tapes, across a steel platform. There is no lid. a) Determine the normal force. b) Determine the force of static friction. c) Determine the force of kinetic friction. d) How much force would she have to apply to make the barrel start moving? e) How much force is required to move it at a constant velocity?



Example F.9 Calvin is dragging Hobbes by his feet down the hallway. Hobbes has a mass of 80kg and he is pulling Hobbes at a constant velocity of 4m/s while applying a force of 275N. Determine the coefficient of friction.



Example F.10 A Physics student has been assigned the task of determining the coefficient of friction between a piece of plastic and a table top. He built a small sled and tried the following masses i) 500g ii) 2kg iii) 5kg. For the first mass he pulled the sled at a constant velocity and measured his force to be 1N. For the second mass the force required was 3.9N and for the third it was 10.2N. Determine the coefficient of friction for each case. You can assume that the experiment was conducted at a constant velocity.

Example F.11 Nadine is dragging a large crate across her basement floor. The weight of the crate is 980N. a) If the coefficient of static friction between the floor and the crate is 0.4, will she be able to pull the crate with 350N. (determine the force of friction) b) Once the crate starts moving determine the net force and the acceleration of the crate if she still pulled with 350N and the coefficient of kinetic friction is 0.18.

Example F.12 Harry is going along in his hemi-powered mud buggy when he hits a Huge mud hole. The force of friction from the mud is 8000N. His buggy can only apply a force of 4000N. If the mass of the buggy is 1000kg determine

- The net force
- The acceleration
- How far the buggy goes before it comes to a stop if he was going 72 km/h.

Example F.13 John was out sliding with his little brother Chevy. The two of them are on a toboggan and their total mass is 140kg. As they hit the flat part at the bottom they hit a long patch of asphalt. At the start of the pavement they were going 20m/s and at the end of the 50m stretch they are going 2m/s.

- Determine the acceleration
- Determine the net force
- Determine the frictional force
- Determine the coefficient of kinetic friction



Example F.14 Zakk is on his motorized scooter when he decides to accelerate. If he accelerates with 3.5m/s^2 (which is the maximum) and the mass of the scooter and rider is 200kg determine:

- The net force
- The friction force is about 200N. Determine much force the engine applies to achieve this acceleration.



Example F.15 K-Zed accelerates from a velocity of 5m/s to 15m/s in 4 seconds. If the mass of the sled and her is 150kg and the coefficient of friction is 0.25 determine the amount of rocket force that provides this acceleration.

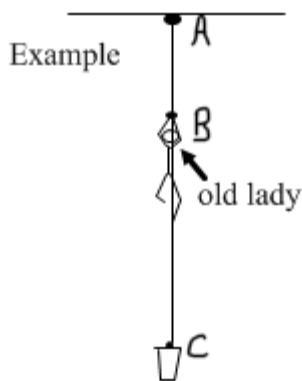


Forces and Vertical Motion

In addition to horizontal motion we need to look at the forces involved when objects move up or down.

Some important points we need to know/remember

1. If the acceleration of the object is upward it will be positive
2. If the acceleration of the object is downward it will be negative
3. Tension in a cable/rope/string is equal everywhere between points of contact



•Tension is equal at all places in the rope between A and B

•Tension is equal at all places between B and C

4. If the mass is not moving or is moving at a constant velocity, the force of tension in the cable is equal to the weight. (This is the same principle we used for horizontal motion)

5. If the objects accelerate upward or downward we can solve by determining the net force.

For any object moving in a vertical plane there are only two basic forces we are concerned with:

1. F_g -Force of gravity - which acts downward
2. F_T - Force of tension - force in the cable which is provided by any force that allows the object to move upward or downward(without free fall)

If we look at the FBD above we can create an equation for the net force



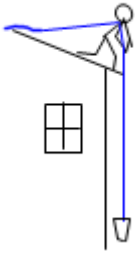
$$F_{\text{net}} = F_T - F_g$$

F_{net} = net force (N)

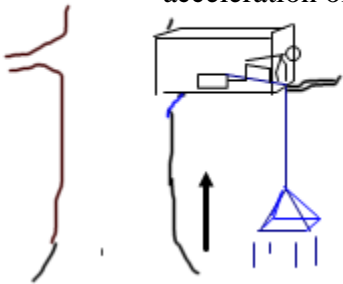
F_T = force of tension (N)

F_g = weight (N)

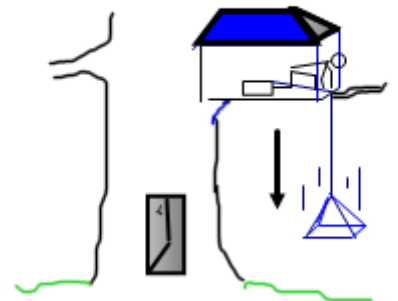
Example F.16 Jasper is trying to lower a pail of nails from the roof of his house by using a rope. The pail has a mass of 30kg. Determine the tension in the rope if the pail is lowered at a constant velocity.



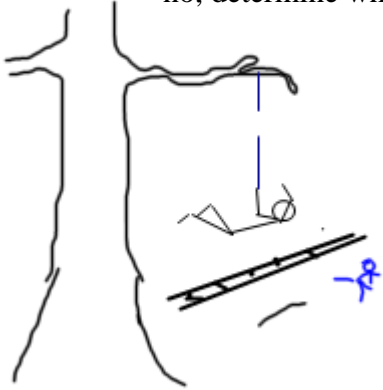
Example F.17 Gregory is pulling supplies up to his tree house using a rope and a little motor that his dad built him. If the mass of the supplies is 40kg and the motor can provide an acceleration of 2m/s^2 , determine the tension in the rope.



Example F.18 Gregory is now trying to lower the supply box and some extra stuff down to the ground. The weight of the box and stuff is 392N. Determine the force of tension in the rope if the acceleration is 3m/s^2 downward.



Example F.19 If John's rope, in an example similar to the one above, can hold 300N can he lower himself down at a constant rate if his mass is 45kg? He can't get down without the rope because it is too high and the squirrels knocked over his ladder. If yes, support your answer. If no, determine what he should do.



Example F.20 A large electric motor is used to lift a 400kg crate of peanuts to the top of a 20m high shelf. The force of tension in the cable during the acceleration is 4045N.

- Determine the acceleration.
- Determine how long it takes to reach the shelf if it accelerates for $\frac{3}{4}$ of a second and travels at a constant velocity the rest of the way.

Example F.21 Haylee is tired of listening to Brooklyn's derogatory remarks so she puts her in a small wooden box and then hooks the box to a cable. The cable goes over a pulley. She pulls on the cable with a force of 800N. If the box and Brooklyn has a mass of 65kg determine the acceleration of the box. b) Jade climbs up to break Brooklyn out. If the cable can hold 1000N what is the maximum mass Jade can have and not break the rope? Assume Haylee fixed the rope so it wasn't going to move.

Newton's 3rd Law

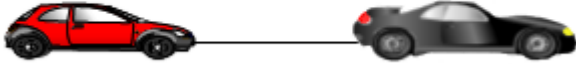
Example F.22 Carl walks down the hallway quickly (in the positive direction) when he runs head first into his school crush (Elvira). His immediate acceleration is -3m/s^2 . a) If his mass is 60kg determine the force that she applies to him to slow him down. b) What is her acceleration if her mass is 40kg . Draw FBDs for each person.

Example F.23 Hugo accidentally drives his 1530kg Corvair into the back of a 10000kg Mack dump truck. The car goes from 40m/s to 0m/s in 1.5seconds . a) Find the force the truck puts on the car. b) What is the force the car puts on the truck c) What is the acceleration of the dump truck. Assume no friction.



Example F.24 Mark's car is broken down on the side of the road. Harold has offered to tow him home. Mark's car has a mass of 1000kg and the coefficient of friction on both cars is 0.25. Harold's car has a mass of 1500kg. Determine the following things ($a = 1.5\text{m/s}^2$):

- force on the hitch on Harold's car
- force the pavement puts on the tires of Harold's car to allow him to pull Mark (the force the pavement applies is a horizontal force to allow the tire to turn and move forward and not spin)



Example F.25 Aaron is baling hay for his grandfather. a) Determine the force in each hitch based on the following information. The tractor has a mass of 5000kg, the baler has a mass of 2000kg and the wagon has a mass of 2500kg. The acceleration of the wagon is 1.2m/s^2 . The coefficient of friction is 0.2. b) Find the force the ground applies to the car to allow the tractor to move forward.



Example F.26 A train is hauling 2 boxcars. Each boxcar has a mass of 6000kg and the mass of the train is 10000kg. The force on the last hitch hauling the last boxcar is 16560N. The coefficient of friction for all the boxcars and the train is 0.20

- a) Find acceleration of the train and the boxcars
- b) Force on each hitch
- c) Force the train wheels put on the track (horizontal)

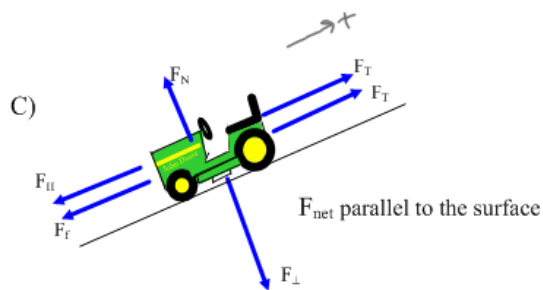
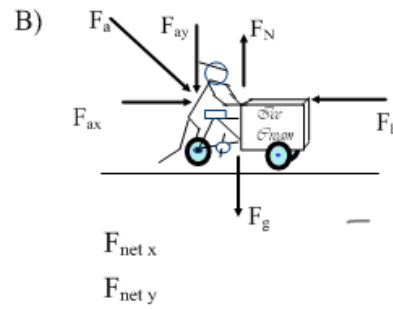
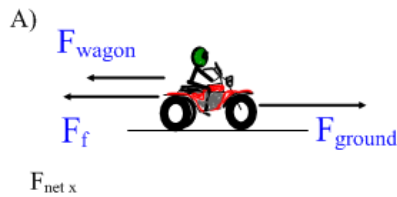


Example F.27 Theodore is driving his car when he runs into the back of a parked 70's style van with shag carpeting. The van has a mass of 2200kg and as a result of the collision has a velocity of 20m/s within 1 sec. a) Determine the force that the car put on the van. b) Determine the acceleration of the car if its mass is 1240kg. The coefficient of friction was determined to be 0.5.

Example F.28 Bob runs his Corolla into Jenny's civic. The force that Bob experiences is 20000N. If the corolla has a mass of 1800kg and the civic weighs 13720N, determine their accelerations. To simplify the situation you can assume that neither driver hit the brakes and friction was negligible.

Example F.29 Adam is driving his father's jag when he got it stuck. (Don't tell Ted) He calls Todd to pull him out. Shaun called to make fun of him. a) Determine the force on the hitch on Todd's dodge if he pulls Adam out with an acceleration of 2.75m/s^2 . The mass of the dodge is 2300kg and the jag has a mass of 1800kg . b) Determine the force of tension in the rope that connects them. c) Find the force the ground puts on the tires.

FBD and F_{net} practice problems



Create FBDs for the following situations:

- Car skidding to a stop
- Pushing a shopping cart with constant velocity.
- Trying, unsuccessfully to push a car.
- Accelerating a mass upward
- Accelerating a mass downward

Terminal Velocity

Terminal velocity is the final velocity that an object can reach and cannot exceed, usually due to air resistance. Unlike surface friction, which is dependent only on the coefficient of friction and the normal force, air resistance increases as speed increases.

As an object travels through the air the force of air resistance will increase until it is equal with the force that is driving the object through the air. The surface area of the object is a large factor in an object's terminal velocity.

For an object in free fall it will reach terminal velocity when the force of air resistance is equal to the object's _____

For an object moving horizontally the terminal velocity depends on the force applied by the engine.

Momentum and Impulse

Momentum is the mass of an object multiplied by its velocity. It is expressed as **p**.

$$p = mv$$

p = momentum (kg m/s)

m = mass (kg)

v = velocity (m/s)

Example F.30 Arnold is travelling at 5m/s with a mass of 80kg. Brianne is running at 7m/s and she has a mass of 65kg. Who has more momentum?

Example F.31 How fast would Arnold need to travel to have the same momentum as Brianne based on the information in the previous example.

Impulse

Impulse is the product of the average force multiplied by the time interval of the force.

You can think of impulse as a quick application of force or a "pulse". We often use it with reference to collisions or sudden changes in force.

$$J = Ft$$

J = impulse (kg m/s or N s)

F = average force (N)

t = time interval (s)

Example F.32 Marshall feels an impulse of 140kg m/s when he bumps into the door frame. How long was the impulse if he felt an average force of 260N?

Impulse-Momentum Theorem

During a crash it is very hard to measure the average force felt during impact so another approach was developed. It says that the impulse is also equal to the change in momentum. This is actually the original version of Newton's Second Law!

$$J = \Delta p$$

J = impulse

p = momentum

Or

$$J = mv - mv_0$$

J = impulse (Ns or kg m/s)

m = mass (kg)

v = final velocity (m/s)

v₀ = initial velocity (m/s)

Example F.33 Allison is driving her 1982 BMW 5 Series Sedan when she hits an ice patch, loses control and crashes head first into a wall. Her velocity just before she collided with the wall was 20m/s and she bounces off the wall with a velocity of 4m/s. Determine, given that the mass is 1200kg:

a) the impulse

b) the average force if the time of the impact was 0.8seconds

Example F.34 Jordanna is driving her car when she collides with a parked truck. If the total mass of her car is 1400kg and she feels -105 000N of force, determine her initial velocity given that the collision took 0.6 seconds and the final velocity was -4m/s. Use impulse-momentum method.

Example F.35 A test car experiences a force of -300 000N over a 0.4 second interval. The initial velocity was 60m/s and the final velocity was -5m/s. What is the mass of the car? What is the acceleration of car?

Conservation of Momentum

system - in physics, the interaction between 2 or more objects

Law of Conservation of Momentum - states that all of the momentum present in a system before a collision or explosion is equal to all the momentum present after a collision.

Situations involving conservation of momentum can be broken into 3 main categories:

1. **Elastic collisions** - objects are apart before the collision and bounce off each other and therefore stay apart after the collision
2. **Inelastic collisions** - objects are separate before the collision and locked together after
3. **Explosions/Separations** - the object(s) is together before the explosion and is in 2 or more parts after the explosion. (Basically the reverse of inelastic collisions)

Elastic Collisions

$$m_a v_a + m_b v_b = m_a v_a' + m_b v_b'$$

m_a = mass of first object (kg)

m_b = mass of second object (kg)

v_a = velocity of first object (m/s)

v_b = velocity of second object (m/s)

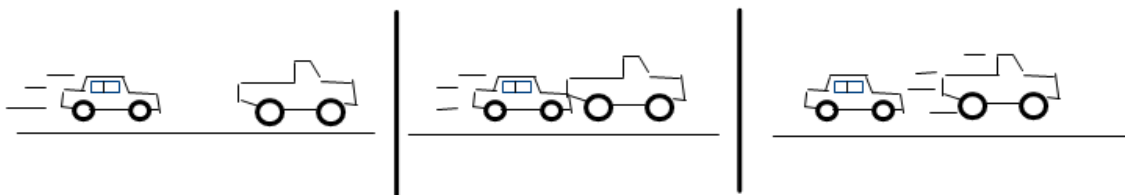
v_a' = velocity of first object after the collision (m/s)

v_b' = velocity of second object after the collision (m/s)

' = signifies after the collision/explosion (prime)

*Note we do not have to use a and b to represent objects 1 and 2. We can use some symbol that makes it easier to keep them separate. For example if a car is in the question you can use m_c to represent mass of the car.

Example F.36 Matt is travelling at 20m/s in his 800kg white 4 door sedan. He rearends Alex's slow moving 1987 white dodge pick up truck which is moving in the same direction at 3m/s with a mass of 1200kg. If Alex's truck has a velocity of 16m/s after the collision determine Matt's velocity after the collision.



Example F.37 Liam is driving his red 89 Toyota Tercel at 50km/h east on a cloudy Thursday afternoon when he is in a collision with Steve. Steve is driving his faded yellow Nissan pick up truck at a velocity of 42km/h in the opposite direction. If the Tercel has a mass of 1100kg and the Nissan has a mass of 1400kg determine the velocity of the Nissan if the velocity of Liam's Tercel is -20km/h after the collision. What impulse is felt by the Nissan?

Inelastic Collisions

$$m_a v_a + m_b v_b = (m_a + m_b) v'$$

v' = velocity of the objects once they are locked together

Example F.38 It was a cold, slippery winter day in February. Campy is driving 72km/h in his titanium silver, 690kg Civic coupe. Up ahead around the corner, in the middle of the road, is Ted putting along in his green, 1600kg Buick Century. Suddenly, as he rounds the corner, Campy sees Ted and smashes into him and the two cars lock together in a tangled mass of twisted metal. If their velocity after the collision was 8m/s how fast was Ted travelling before the incident?

Explosions/Separations

$$(m_A + m_B)v = m_A v_A' + m_B v_B' \quad v = \text{velocity combined before}$$

Example F.39 A 12kg bomb is placed in a safe location so as not to injure any one(this is an experiment). The bomb explodes into 2 separate pieces. The first piece flies directly east at 80km/h and has a mass of 9kg. What is the velocity of the other piece?

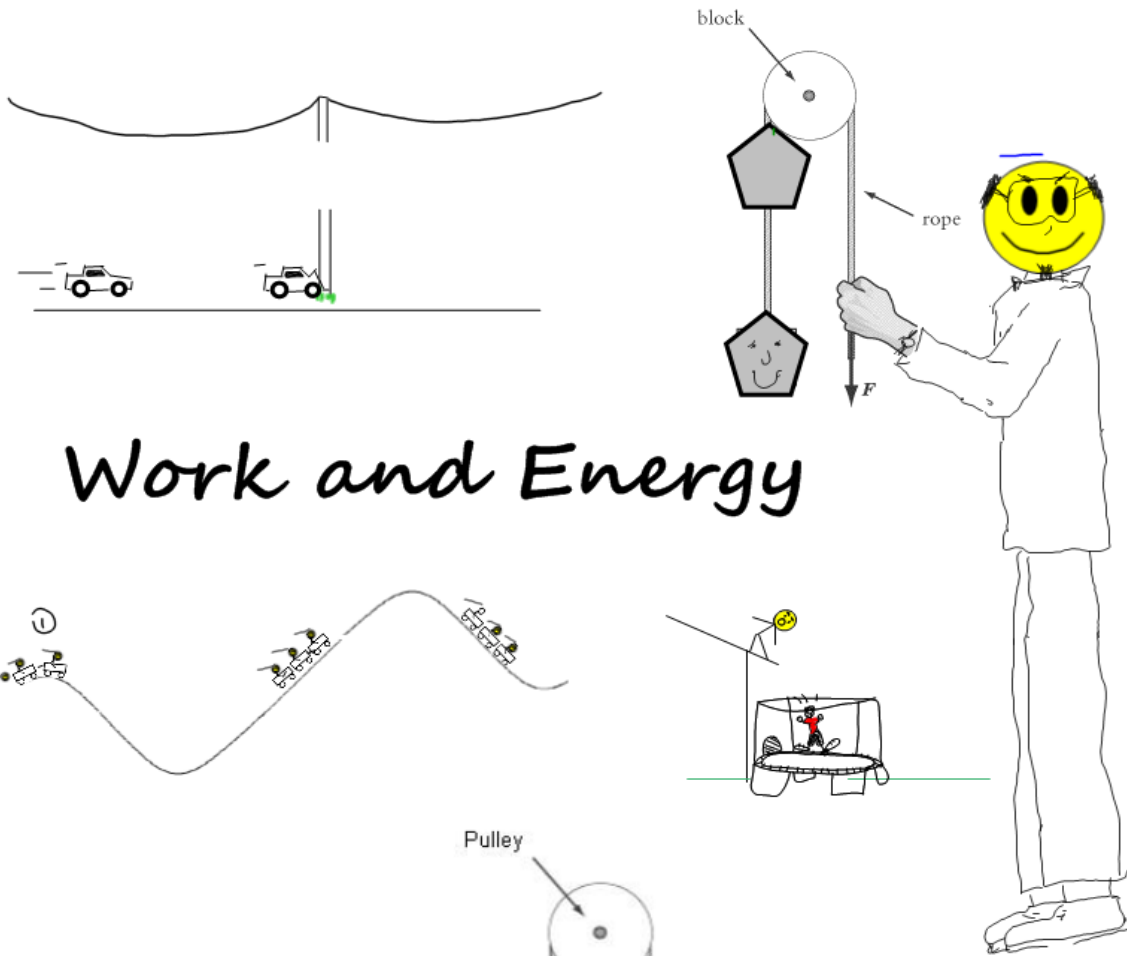
Example F.40 **Bobby Jo** is experimenting with her remote control robot missile launcher. The robot and launcher as one unit and has a mass of 50kg and is travelling at 4m/s. The missile has a mass of 500g. What is the velocity of the robot and launcher after if the velocity of the missile is 250m/s when it is shot? (The missile is in the launcher before it is shot. What a coincidence. No robots were harmed during this example)

Example F.41 Shammy is bored and decides to go shoot at wooden blocks in his back yard. He and the rifle have a mass of 85kg. After shooting the block, it and the bullet fly off at 4m/s. The block has a mass of 1kg and the bullet has a mass of 25g. a) Determine the velocity of the bullet before it hits the block. b) Determine Shammy's velocity after the bullet leaves the rifle.

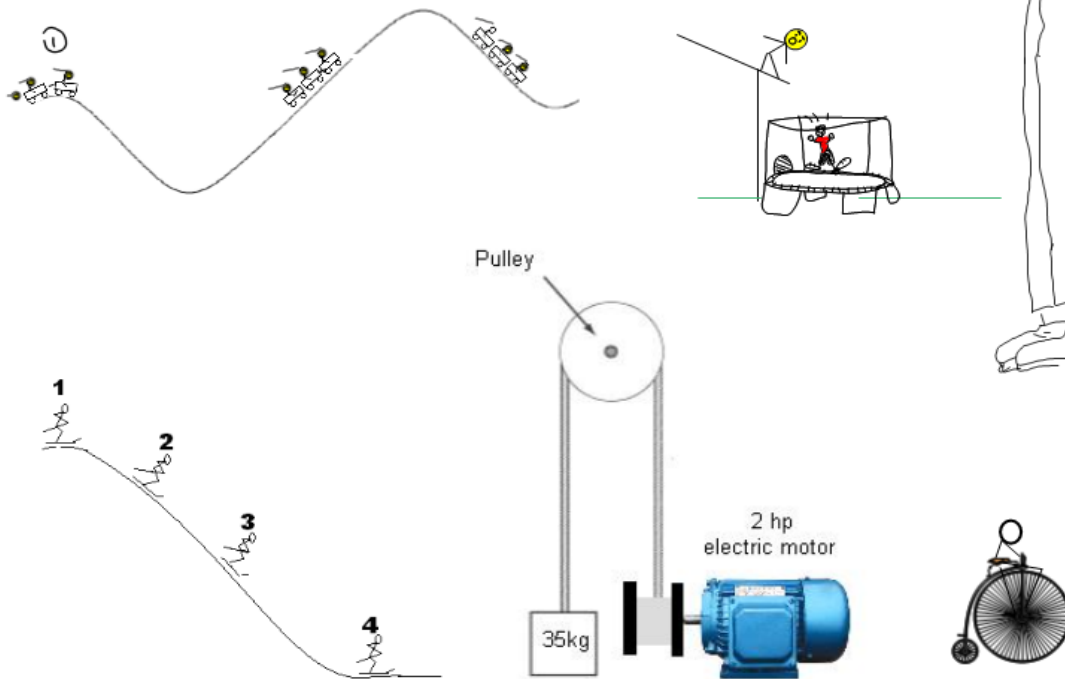
Example F.42 Andy is involved in the following 3 car accident with Bob and Charlotte(aka Campy). Andy's car has a mass of 1300kg and is travelling at 80km/h when he rear ends Bob who is parked at a stop sign. Bob's car has a mass of 1200kg. After the collision Andy slows to 20km/h. Bob then moves forward and crashes into Campy who is parked in front of Bob. Campy's car has a mass of 1000kg. If Bob comes to a stop again after the collision determine

- His velocity after been hit by Andy
- Campy's velocity after being hit by Bob.
- How much energy is lost in each collision.





Work and Energy



Unit 3 - Work And Energy

Work and energy are related. For example: You do work by pushing a box down a hallway. As a result of the friction heat energy is given off. Or you as a person will use energy to do work.

Work - is the force acting on an object multiplied by the displacement the object is moved. The direction of the force and the displacement must be parallel.

$$W = Fd$$

W = work (J- joules)

F = force (N)

d = displacement (m)*

*must be parallel to direction of force

Example E.1 How much work is required to push a blue box a distance of 12m with a force of 1200N?

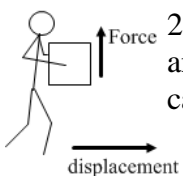
Example E.2 Melvin is putting 40kg bags of wood pellets on a shelf that is 1.6m high. If he lifts up 10 bags how much work does he do?

*The force may not always be stated in the question. If an object is being lifted it (the force) is rarely stated, only the mass of the object will be stated. In this case we usually assume the object will be picked up with a constant velocity therefore the lifter would have to supply a force equal to force of gravity.

There are certain instances in Physics where a force is being supplied but no work is being done.

1. A force is applied but the object is not moving. Since there is no displacement, there is no work being done. Ex. You push on a door but it won't open.

2. The force being applied is perpendicular to the displacement. Objects can move at an angle and work can be done but when the force is 90° there is no work being done. Ex. You are carrying a box as you walk down a hallway.



Example E.3 Hakim has been asked by his boss to carry boxes of frozen food down the hall and stack them on a 2m high shelf. To do this he picks up each box from the floor to a height of 1.2m. He then walks 20m to the end of the hall and then puts the box on the shelf which is 2.0m high. Determine the work done on a 10kg box.

Energy - in Physics terms it is the capacity for doing work. For example an object sitting on a shelf has stored energy which may be converted to do work when it falls.

There are many types of energy:

- kinetic*
- potential (gravitational*, elastic*, chemical)
- electrical
- sound
- thermal

Everything has energy. In general terms anything in motion has kinetic energy. Anything at rest has potential energy. Objects may have more than one type of energy at any given point in time.

Kinetic Energy

It is the energy of motion. When an object is moving it has kinetic energy.

$$E_K = \frac{1}{2}mv^2$$

E_K = kinetic energy (J)

m = mass (kg)

v = velocity (m/s)

Example E.4 Jordan is out for a ride on his unicycle. If he is travelling at 5.0m/s and he has a mass of 40.0kg determine his kinetic energy.



Example E.5 Kim has 7500J of energy while biking on her penny-farthing . If his mass (with the bike) is 65kg determine his velocity.



Example E.6 Natalia has a mass of 500kg and a velocity of 72km/h. a) Determine her kinetic energy. b) Her friend is travelling at 90km/h and has the same amount of energy determine her mass. (Note: they are motorized bicycles)

Gravitational Potential Energy

Is the energy stored in an object when it is placed at a position higher than the starting point. We have to choose a reference point. A reference point is usually the ground. The reason for a reference point is that the potential energy of a book on a shelf is different with reference to the floor than it is to the ground.

$$E_g = mgh$$

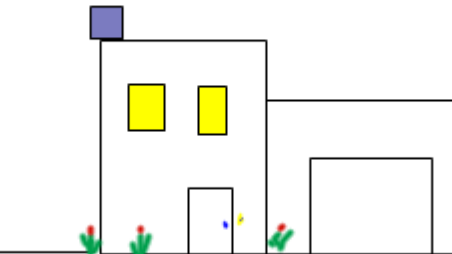
E_g = gravitational potential energy (J)

h = height above reference point (m)

g = gravity (9.8m/s^2) (positive in this situation)

m = mass (kg)

Example E.7 Determine the amount of gravitational potential energy of a 100kg crate sitting on a roof of a 18m high building. (Our reference point is the ground in this situation).



Example E.8 A bowling ball is sitting on a rack, 1.25m above the floor. It has a potential energy of 65 joules. Determine the mass of the bowling ball.



Elastic Potential Energy

Energy is stored in a spring when it is compressed or extended. Like gravitational potential energy we need to use a reference point, for springs we call it the rest position. Understanding energy in springs is an important concept in designing suspensions in cars, bathroom scales.

$$E_E = \frac{1}{2} kx^2$$

E_E = elastic potential energy (J)

k = Spring Constant (N/m)

x = Extension or compression of the spring
(Measured as a distance from the spring's rest position) (m)

Example E.9 Determine the amount of energy stored in a spring that is compressed 40 cm past its resting point if the spring constant is 300N/m.

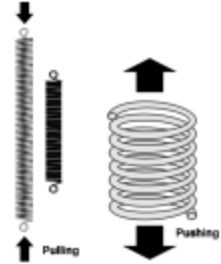
Determining the spring constant

The applied force on a spring is directly proportional to the extension or compression of the spring. This is called Hooke's law.

$$F_A = -kx$$

F_A = applied force (N) *may be applied by weight on an object or when you pull or push on a spring
 k = spring constant (N/m)
 x = extension or compression (m)

The equation has a negative due to Newton's Law. Where the force applied to the spring is equal to the force from the spring only in the opposite direction. In the diagram to the right the arrows indicate the direction of the force from the spring not the applied force.



Example E.10 Kevin has a mass of 80kg and stands on a spring and compresses it 25cm past the rest position. a) Find the spring constant. b) Find the energy in the spring (Hint - the applied force is his weight.)

Summary Of Types of Energy

All three types of energy we have discussed are considered mechanical energies

Kinetic Energy

E_k = object is moving (formula)

Potential energy

E_g = object has height (formula)

E_e = involved a spring (formula + spring constant form (2 forms)

Quiz

State the type of energy for each situation below. Your options are:

Kinetic Energy - E_k

Gravitational Energy - E_g

Elastic Energy - E_e

A combination of the above

Another type of energy we haven't discussed

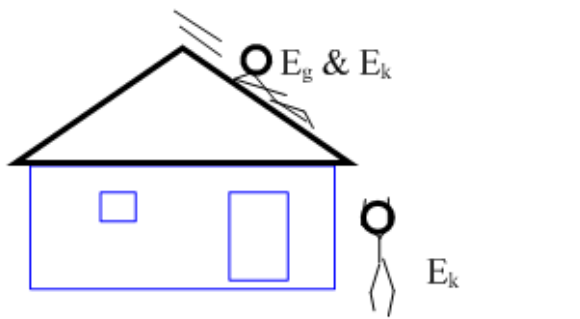
1. A box sits on a shelf
2. Albert hits a golf ball and it flies through the air
3. Billy jumps on a trampoline
4. Celia stands at the top of a ski hill
5. David is sliding down the roof on a house
6. Evan is on a roller coaster that is going up the steep side of the roller coaster track
7. Frank is running with a large rubber band attached to his back that is starting to stretch
8. Gina throws a ball in the air and it reaches its maximum height
9. Hank falls while running and is sliding to a stop
10. Ingrid runs her bumper car into a wall and comes to a complete (but momentary) stop
11. Jack's stove is turned on and is boiling water
12. Kelly's sliding down a hill on her sliding apparatus (a.k.a a cardboard box)

Sketches/Diagrams

At higher levels of education there is a large importance placed on diagrams and sketches to help solve higher level problems. As such it is required in this course that you draw a sketch/diagram and properly label it whenever you are using Conservation of Energy or, as already discussed, Forces. The expectations of a diagram are to show an idea of the situation and the types of energy that exist at certain locations as discussed in the question.

Here are a couple examples:

David is sliding down the roof of his house. What is his velocity when he hits the ground? *Note : remember he has velocity when he hits the ground or it wouldn't hurt!



Gino throws a football in the air from waist height. Determine the maximum height.



Conservation of Energy

All energy in the world cannot be created or destroyed. It can only be changed from one form to another.

The formula for *Conservation of Mechanical Energy* neglects energy losses due to friction. (We will cover that in the near future.) It states that the “total energy in a system is equal at all points in the system.” A system is basically the interval between what we have chosen to be the initial/starting point and the final/after point. These points can be changed to solve for different variables. The Mechanical Energy refers to the types of energies we are using currently.

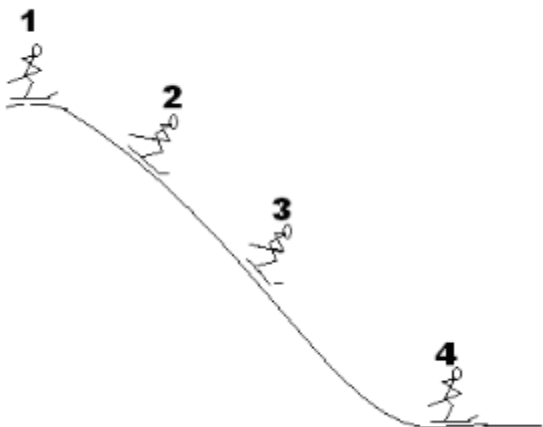
The definition in words would be:

All energy (in a system) before an event = All energy (in the system) after the event

$$E_K + E_G + E_E = E_K' + E_G' + E_E'$$

E_K = kinetic energy (J)
 E_G = gravitational energy (J)
 E_E = elastic energy (J)
' = indicates after or final

*Note: 1 - mass will remain constant throughout the situation
2 - Unless friction is mentioned we will assume that it is 0.



In this diagram we can use any combination of two points to solve for missing data. Each point on the hill has the same amount of total energy but the amount of kinetic and gravitational energy can change.

1. All E_G
2. E_G is decreasing, E_K is increasing
3. E_G is decreasing even more, E_K is increasing even more
4. All E_K

Total mechanical energy remains the same at each point

Example E.11: A skier with a mass of 60 kg starts from rest at the top of a 10m high hill. If there is no friction determine: a) The velocity at a height of 4m. b) The velocity at the bottom of the hill.

Example E.12 A roller coaster is travelling at 4m/s at the top of a 8m high part of the track.
a) Determine the height when the velocity is 2m/s. (We know the mass is 100kg) b) What is the maximum height the coaster could reach?

Example E.13 Rob has a mass of 80kg and is standing on top of a 4m high roof. He jumps off the roof onto a trampoline with a spring constant of 4000N/m. We will assume the trampoline stretches so that there is no gravitational potential energy.

- a) Find the amount the trampoline stretches
- b) How high will he get in the air after?

Example E.14 How far would the trampoline stretch if it was 2m high instead of just the right height to touch the ground?

Example E.15 A van has a mass of 300kg and is parked against a large spring that has a spring constant of 30kN/m. The spring is compressed 2m from the rest position. a) Determine the speed of the van at the foot of the ramp (assume no friction and that the spring is released)
b) How fast is the van moving at the top of the ramp if the length of the ramp is 4m long. (You need to find h)

Bonus: Will the van land in a pond that is 10m(center of the pond) away from the ramp (4m wide)?!



CONSERVATION of TOTAL ENERGY

We are still dealing with the conservation of Mechanical energy but **we are now accounting for work as well**. Work that is done by external forces such as friction to slow us down or by something such as an engine to increase or maintain our speed.

In summary, we can say that the work done in a system is equal to the difference between the initial and final energy.

Work = Total Final Energy - Total Initial Energy OR Change in Energy

$$W = E_f - E_i$$

$$W = \text{Work (J)}$$

$$E_f = (E_G' + E_K' + E_E') \text{ (J)}$$

$$E_i = (E_G + E_K + E_E) \text{ (J)}$$

Review:

Work is equal to the force multiplied by the distance.

$$W = Fd$$

The value of F is the force that is doing work. For example if you are pushing a box across a floor with a constant velocity you are doing work to the box and friction is also doing work to the box. You can calculate different types of work depending on the situation. If we use F_{net} then no work would be done when an object moves at a constant velocity.

The Work in the $W = Fd$ equation is equal to the W in the Energy equation and can be substituted in place of each other

Example E.16 Lester is at the top of a slide that is 4m high. If his speed is 7m/s at the bottom and his mass is 30 kg determine the following:

- A) Work done to slow him down
- B) Force of friction if the ramp is 6m long

Example E.17 Becky is at the top of a roller coaster. Her mass is 100kg and she is 20m high with a velocity of 20m/s. If the work done against her is 5000 joules determine her velocity when she is at a height of 5m.

Example E.18: Carl is travelling at 20m/s at the bottom of a hill. How much work is done if his velocity at the top of a 5m high hill is 2m/s. (His mass is 1000 kg)

Example E.19 Jessica is travelling in her car at 144km/h and not really paying attention. If the front wheels fall off and she skids to a stop determine the following if her mass is 1500kg.

- A) Work done to stop the car
- B) Force of friction (it took 120m to stop the car)

Example E.20 Tom is travelling in his golf cart at the bottom of a hill at 10m/s. How much work would the motor on his gold and green Honda powered golf cart need to do to get to the top of a 6m high hill at 6m/s? If the hill is 20m long how much force is applied by the engine. (Mass of cart is 300kg)

Example E.21 Bella is lifting a 30kg box of vintage staplers. It is lifted from the floor to a 2m high shelf.

- a) How much work is done to lift the box to the shelf
- b) If the box falls off the shelf how fast will it be going when it hits the floor?
- c) How fast would a 60kg box be going when it hit the floor?

Example E.22 The pendulum shown below has a length of 1.2m and the bob has a mass of 2.0kg. If the bob is lifted to a height where the angle to the vertical is 22° determine:

- the potential energy of the bob before it is released
- velocity when it passes through the lowest point of travel
- calculate a & b again if used on a planet with an acceleration of gravity of 2.3m/s^2



Summary

1. Draw a sketch
2. Determine the types of energy that exist at the points in question
3. Determine if work is being done.
4. Choose the appropriate equation. ($W = E_f - E_i$ works for any situation)

Work, Power, & Efficiency

Anything that does work will have a power and a certain efficiency. Engines and motors have power ratings.

Efficiency is directly related to the energy lost to friction. We use oil in engines to reduce friction, thus making them more efficient.

Power is the amount of work done divided by the time it takes to do it.

$$P = \frac{W}{t}$$

P = Power (W - watts)

W = Work (J)

t = Time (s)

Example E.23 A motor has a power rating of 500W. How long would it take to do 250 joules of work?



Example E.24 Jack pushes a large crate with a net force of 200N over a total distance of 40m in 2 minutes. How much power does he perform with?

Horsepower

The common unit to represent the power of engines in North America and most other parts of the world is horsepower (abbreviated as hp).

One hp is equal to lifting 550 pounds to a height of 1 foot in one second. Since we use watts we would like to have a conversion between the two units.

$$1 \text{ hp} = 746 \text{ W}$$

Example E.25 Kendra is running up the stairs to determine her horsepower. Her mass is 60kg and it takes her 4 seconds to run up the stairs to a height of 4.5m.

- A) Find her power in watts
- B) Find her hp

Example E.26 A 300hp car travels 400m in 10 seconds. Determine the following:

- A) The power in watts
- B) The applied force
- C) The F_f if the velocity is constant

Efficiency

Efficiency is the ratio of useful energy output divided by the total energy input, expressed as a percentage.

$$\text{Eff} = [E_o/E_i] \times 100$$

Eff = Efficiency (%)

E_o = Energy Output (J)

E_i = Energy Input (J)

Efficiency can also be the ratio of useful work output divided by the work input (or work done in an ideal situation). An ideal situation is one in which there is no energy loss.

$$\text{Eff} = [W_o/W_i] \times 100$$

Eff = Efficiency (%)

W_o = Work Output (J)

W_i = Work Input (J)

*Note:

The work done is often equal to the energy so the equations can be mixed.

Often efficiency is used in engines that use fuel. The fuel used would be the energy input. Efficiency, being expressed as a percentage can never be more than 100%. We ideally want efficiency of anything to be as close to 100% as possible.

Example E.27 A car stereo uses 360J of electrical energy to produce 270 J of sound energy.

- A) What is the efficiency?
- B) Where has the missing energy gone?

Example E.28 A 1500 kg car (okay, Ice cream truck) starts at rest and speeds up to 3.0m/s

- a) What is the gain in kinetic energy?
- b) If the efficiency is 30%, how much input energy was provided by the gasoline?
- c) If 0.15 gallons were used up in the process, what is the energy content of the gasoline in Joules per gallon?



How to Determine Which Work is Input and Which is Output

W_I is usually to work done when a force is applied over a certain displacement

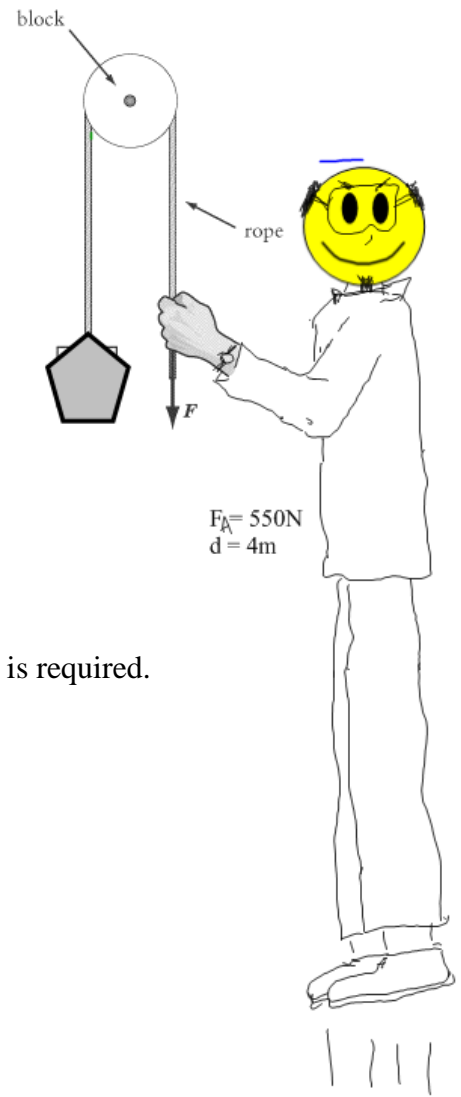
W_o is the ideal work done if no friction is assumed and is simply the difference between the initial and final energy of the object in question.

$$W_I > W_o$$

Example E.29: Walter is using a pulley to lift objects to a 4m high storage rack. In order to lift the 50kg bags he has to apply 550N of force while pulling the 4m of rope.

A) Determine the efficiency

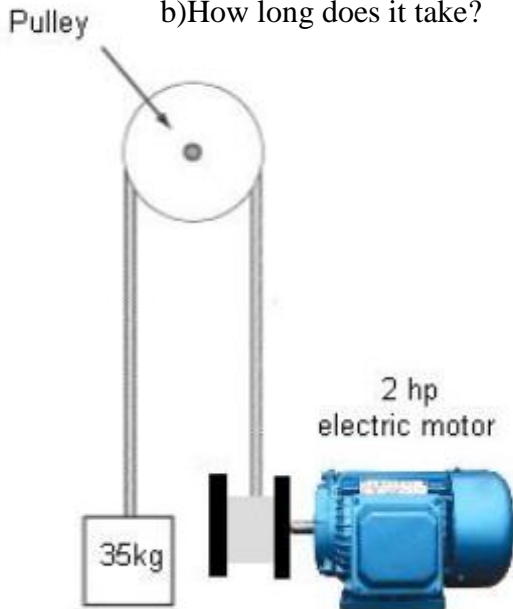
B) Determine his power in watts (i) and hp (ii) if it takes him 5 seconds



Example E.30 A 2hp motor lifts a 35kg mass to a height of 3.5 metres.

a) If the motor has an 82% efficiency determine the electrical energy that is required.

b) How long does it take?

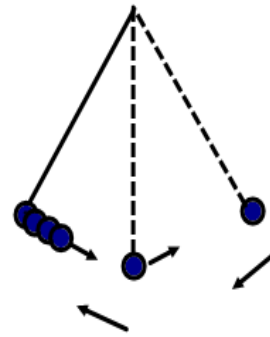
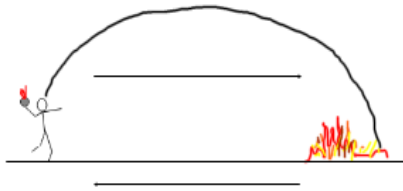


Example E.31 A pile driver's motor expends 0.31 MJ of energy to lift a $5.4 \times 10^3 \text{ kg}$ mass. The motor has been determined to have an efficiency of 13%. How high is the mass lifted?

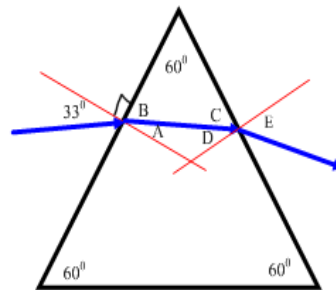
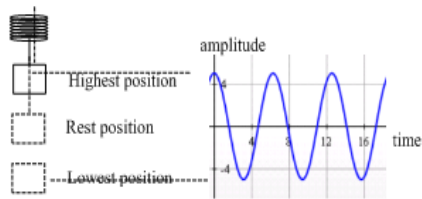
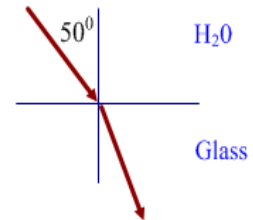


Example E.32 A box is pushed up a ramp that is 10m long and 4m high. If the efficiency of the ramp is 45%, what force would you apply to get the box to the top of the ramp?

Example E.33 A small car with a mass of 100kg starts from rest and attains a velocity of 20 m/s in 5 seconds with a constant acceleration. Create graphs showing position vs. time, velocity vs. time and kinetic energy vs. time.



Waves



Unit 4 - Waves and Energy Transfer

wave - a disturbance of a medium which transports energy through the medium without permanently transporting matter. For example - when you throw a rock in water the energy is transferred to the medium (water) and sends out a "wave" or temporary displacement of the water particles which will return to their original position

- the disturbance may be simply one application of energy such as a pulse (rock thrown in water) or it may be repeatable (ex. wave maker at an water amusement park)

Important Definitions

medium - substance that a wave travels through.

periodic motion - when an object moves in a repeated pattern over regular time intervals
Ex. a pendulum in a grandfather clock swinging back and forth

cycle - one complete repeat of a pattern (also called a vibration)
Ex. If you let a pendulum go, the cycle is completed when it returns to you

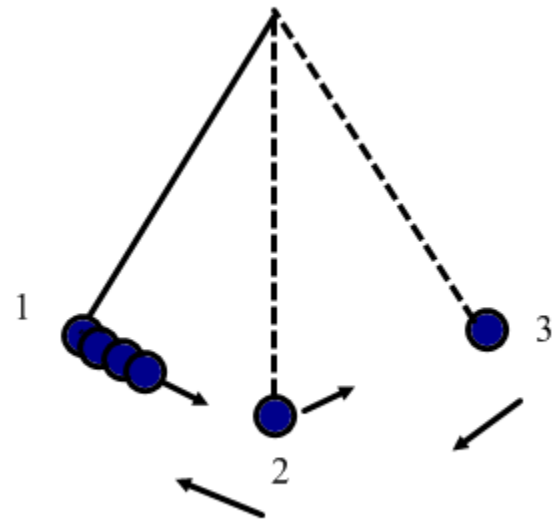
period - time required to complete one full cycle

frequency - number of cycles completed over a given time period. It is usually represented as /s or Hz. We don't include a unit for the number of cycles in the numerator

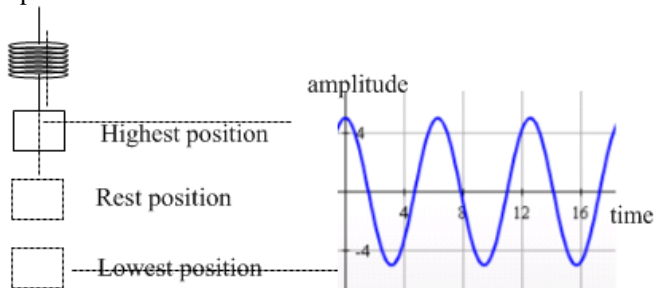
amplitude - distance from the rest position of a vibrating object to its maximum displacement, which may be a vertical or horizontal distance depending on the object

Pendulums and waves

- if the pendulum starts at position 1 it travels through position 2(also the rest position if not moving)
- it will travel up to 3 where it momentarily stops and then returns to 2 then 1
- a cycle is from 1 through to 3 and back to 1
- period is the amount of time it takes to travel this distance
- Regardless of the object there are four distinct parts to a wave as it completes a full cycle.



*the same principles exist for a spring except that it goes up and down instead.



If you were to graph the motion of the spring above as it moved it would look similar to the graph beside it. Seismic graphs work on this principle. A stationary pen draws a line on moving piece of graph paper. When there is an earthquake it sends a wave of energy through the earth which is recorded as a graph similar to the one above.

The pendulum above would create a similar graph as would any spinning object such as a ferris wheel (slow moving), a crankshaft on a motor or a heart monitor.

Determining Period

- if the object is moving/vibrating slowly you could measure each period with a stopwatch
- if it is going faster we can use the formula below

$$T = \frac{\Delta t}{N}$$

T - period (seconds)
 Δt - time interval (seconds)
N - number of cycles (no units)

Example W.1 Through experimentation it is determined that the pendulum makes 40 cycles in 25 seconds. Determine the period.

Example W.2 A spring is bouncing up and down. If it takes 0.40 seconds to complete one cycle, how many cycles would be completed in 2 minutes.

Determining Frequency

- frequency is the number of cycles completed in a given time interval (usually in one second)
- it is the inverse of the period

$$f = \frac{N}{\Delta t}$$

f = frequency (/s or Hz)
N = number of cycles (no unit)
 Δt = time interval (s)

OR

$$f = \frac{1}{T}$$

f = frequency (/s or Hz)
T = period (sec)

Example W.3 A blade on a lawnmower makes 4000 revolutions in 1 minute. Determine a) frequency b) period



Wave Behavior

A *wave* is a disturbance that transfers energy through a medium.

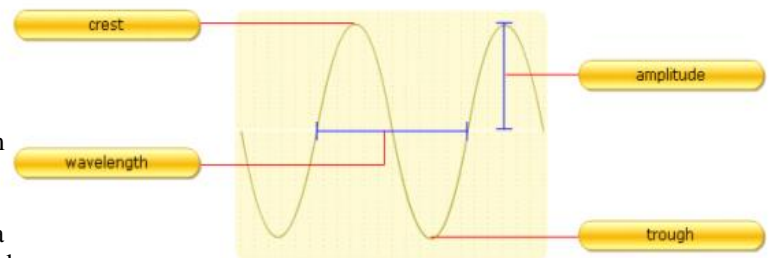
The speed of the wave in a medium is a property of the medium. For example: sound waves move faster in water than they do through air. However, not all waves require a medium and as such waves can be broken down into 2 main categories.

Two types of waves

1. **mechanical** - require a medium such water waves, sound waves, waves in strings, slinkies, etc
2. **electromagnetic** - do not require a medium, they can travel through a vacuum such as you would encounter in space
- examples would be light waves, radio and TV waves and microwaves

Parts of a Wave

1. **crest** - is the highest point on a wave
2. **trough** - is the lowest point on a wave
3. **amplitude** - distance from rest position to maximum displacement (crest or trough)
4. **wavelength** - distance between matching points on a wave such as between crests or between troughs
- we use λ (lambda) to represent wavelength
5. **frequency** - number of waves that pass a given point each second



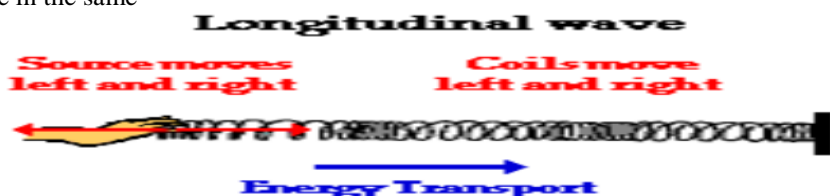
Mechanical Waves

Mechanical waves can be separated into two categories by the way that they cause the particles of the medium to move.

1. **Transverse Waves** - particles of the medium move at right angles to the direction of the motion of the wave



2. **Longitudinal Waves** - particles of the medium move in the same direction as the motion of the wave



Wave Equation

The velocity of a wave is equal to the product of its wavelength and its frequency

$$v = f\lambda$$

v = velocity (m/s)

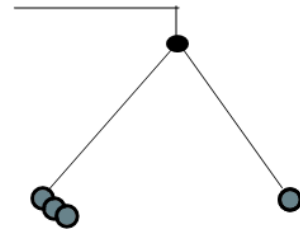
f = frequency (/s or Hz)

λ = wavelength (m)

Example W.4 A wave machine at an amusement park makes waves that have a velocity of 2.8m/s and the wavelength is 5.6m. Determine a) the frequency b) the period.

Example W.5 Marvin is playing around with his guitar. He plucks one string on his guitar and generates a frequency of 512Hz. If the speed of sound in air at that time is 346m/s, determine the wavelength of the sound wave.

Example W.6 A pendulum travels from point A to point B and back 30 times in 1 minute. If the wavelength is 50cm determine the velocity of the wave.



Finding distance travelled with a constant velocity

For our course we will be using the following formula:

$$x = x_0 + vt$$

x = final position (m)

x_0 = initial position (m)

v = average or constant velocity (m/s)

t = time interval (s)

$d = x - x_0$ (m)

In many situations x_0 will be equal to zero but we must state it in our givens and in the equation. When x_0 is equal to zero the distance travelled is equal to the value of x .

Example W.7 Tsunamis are fast moving ocean waves. One particular tsunami travelled 3250km in 4.6 hours. What is the frequency of the wave if the wavelength is 640km.

Waves at Boundaries

When waves move from one medium to another its frequency remains the same.
The speed and wavelength change

There are 2 basic situations

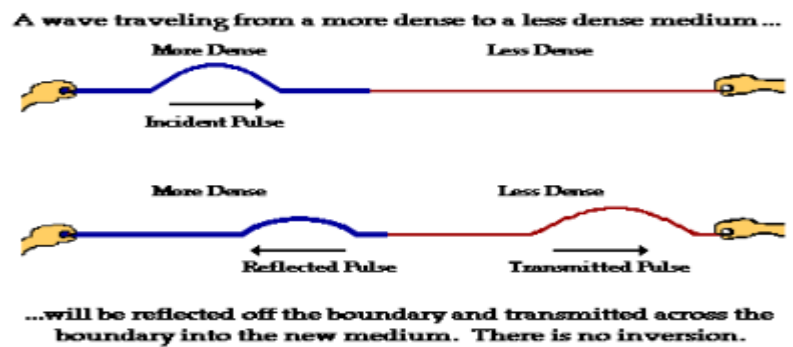
1. From heavy (more dense) medium to a light (less dense) medium
2. From light medium to a heavy medium

* Waves move slow in heavy mediums and faster in light mediums

* Think of a wave in a skipping rope vs. a wave in a chain

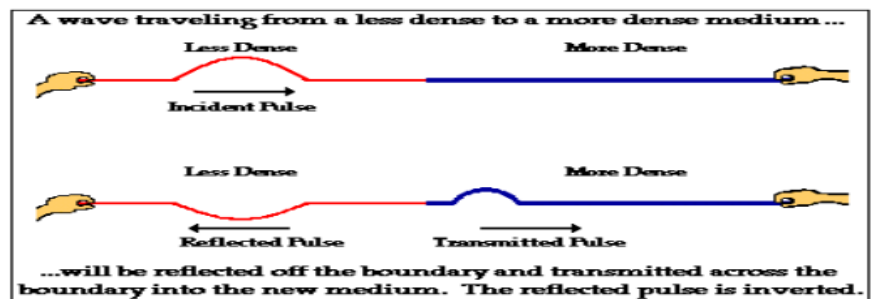
1. Heavy to Light

- part of the wave is transmitted and part is reflected back
- velocity increases
- frequency remains the same
- wavelength increases
- transmitted wave is erect
- reflected wave is erect



2. Light to Heavy

- part of the wave is transmitted and part is reflected back
- velocity decreases
- frequency remains the same
- wavelength decreases
- transmitted wave is erect
- reflected wave is inverted



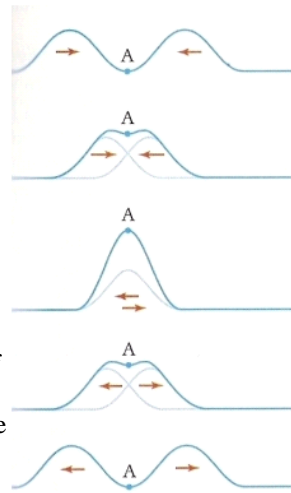
The light to heavy situation also applies to waves hitting a solid structure. The solid structure is the same as a wave moving to a heavy medium, except you can't see the wave that is transmitted.

Superposition of Waves

Occurs when two waves collide. The result is a combination of their amplitudes. There are two basic situations:

1. **Constructive interference**

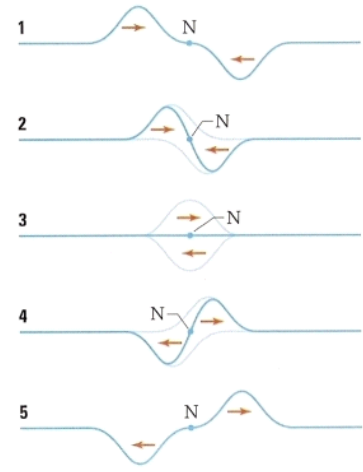
- occurs when two upright waves meet each other
- when they meet the result is one wave that has an amplitude equal to the combination of the amplitudes of the two individual waves



Constructive interference

2. **Destructive interference**

- occurs when one upright wave meets an inverted wave
- when they meet the result is one wave that has an amplitude equal to the combination of the amplitudes of the two individual waves
- if the amplitudes are the same the result will appear to be flat or have zero amplitude

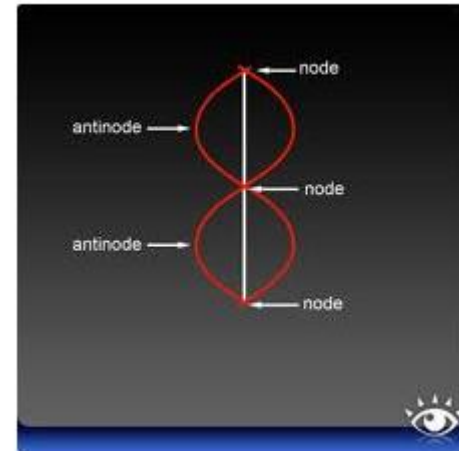


Destructive interference

Standing Waves

- when waves of the same shape, amplitude and wavelength travel in opposite directions in a linear medium such as a rope or spring they will produce a pattern that appears to be standing still
- at intervals of $\lambda/2$ (half a wavelength) there is destructive interference which cause *nodes* (position of zero amplitude)
- the ends of the wave are also *nodes*
- between each node is an *antinode* (position of maximum amplitude)
- standing waves can be created with a source at each end or by a source at one end and its reflection

*Note: a standing wave can be in a string that is fixed at both ends or in a medium that is fixed only at one end such as in a hand saw



natural frequency - frequency that medium will vibrate at when allowed to vibrate freely. For example if you allowed a pendulum to swing on its own it will swing at a specific frequency, a string on an instrument will also do the same thing

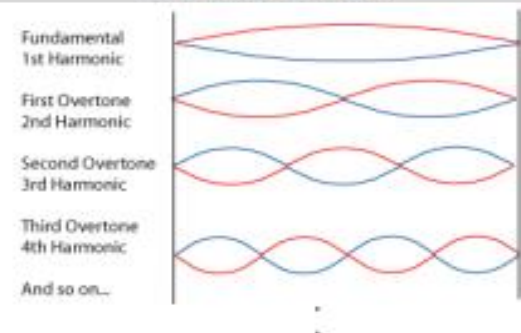
resonance - occurs when energy is added to a vibrating system at the same frequency as its natural frequency
- the amplitudes of the vibrations may become very large

Real World Example: A music box works on the principle of natural frequencies. Each note is generated when a small bump on a wheel makes a small strip of metal vibrate. Each strip is cut to a specific length to achieve a note and the wheel has the bumps in a certain order to create the music

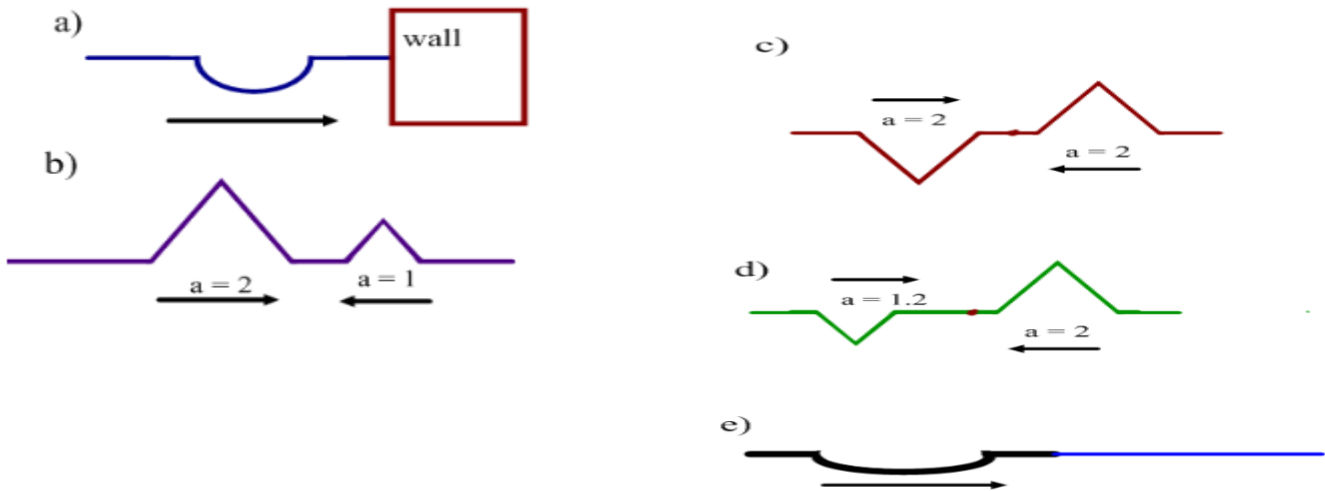
Natural Frequency

- in a medium that has a fixed length there are several natural frequencies that will produce resonance (the medium may be fixed at one end or both)
- the lowest frequency that will produce resonance is called the **fundamental frequency** and this has the longest wavelength
- each natural frequency above the fundamental frequency is called an **overtone**
- each overtone has one more node than the previous one

Standing Wave Harmonics



Example W.8 Draw what would happen in the following questions



Waves in Strings and Musical Instruments

The speed of a wave in a string of an instrument is dependent upon two things:

1. tension of the string - greater the tension the greater the speed
2. mass per unit length - greater the mass the slower the speed

For example: On a guitar the strings have relatively high tension and as a result they have high velocities and high frequencies. A bass on the other hand has thick strings with a higher mass per unit length which results in a lower speed and thus a lower frequency

Speed of Sound in Air

The speed of any sound in air travels at the same speed and is only dependent on the temperature.

The following formula is used to determine the velocity (or temperature)

$v = 331 + 0.6T$	$v =$ velocity of sound (m/s) $T =$ temperature ($^{\circ}\text{C}$)
------------------	---

Converting from Celcius to Fahrenheit $^{\circ}\text{C} \times 9/5 + 32 = ^{\circ}\text{F}$	Converting from Fahrenheit to Celcius $(^{\circ}\text{F} - 32) \times 5/9 = ^{\circ}\text{C}$
Converting from Celcius to Kelvin $^{\circ}\text{C} + 273 = \text{K}$	

Example W.9 Determine the speed of sound in air when the temperature is a) 25°C b) -24°C c) 100°F d) 263K

Example W.10 Determine the temperature when the speed of sound in air is a) 353m/s b) 321m/s ?

Example W.11 Determine the wavelength of a sound that has a frequency of 300Hz when the temperature is -10°C .

Example W.12 Farmer Ralph is outstanding in his field on a cold February morning with temp of -15°C when he sees a tractor start back at the homestead (by the puff of smoke). He hears the sound of the tractor starting 4 seconds later. How far is he from the homestead?



Echoes

An echo is created when a sound wave bounces off a reflecting surface and returns to the source or someone who heard the source.

If you want to determine how far a reflecting surface is you can do one of the following:

1. Find the distance sound travels and divide it in half
2. Divide the time it takes to hear the echo in half and find the distance

If you know the distance to the reflecting surface and want to determine the time it takes to hear the echo you can do the following:

1. Double the distance and find the time
2. Find the time to the reflecting surface and double the time.

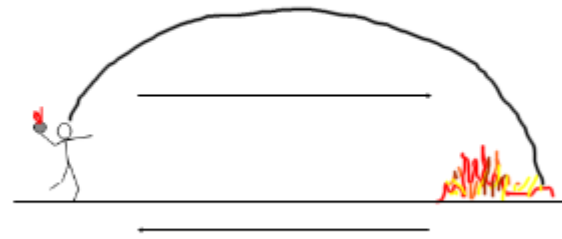
Example W.13 It is a brisk, cool day in December and the temperature is -5°C . Your dog is outside and you need to call him in. You yell his name (Dee Oh Gee) and you hear your echo 2.5 seconds. How far away is the reflecting surface?

Example W.14 Determine the temperature if you are standing 514.5m away from a wall and you hear your echo 3 seconds after you yelled.

Example W.15 On a cool, 50°F day in October, Jennifer decides to do a bit of experimentation with the sound stuff she learned in Physics class. She goes outside of her school and walks a measured distance of 500m from the building. She then claps her hands and hears an echo. How long did it take for her to hear her echo?

Two Part Questions – Something happens that makes a sound and the sound comes back to an observer

Example W.16 Harry Jr. is out with his buddies, raising a little heck. They have concocted a molotov cocktail and decide to throw it into an abandoned parking lot on the south side of town. Harry throws the molotov cocktail with a velocity of 20m/s and it lands 80m away, exploding on impact. How long after he throws it does he hear the explosion? (Temperature is 10°C)

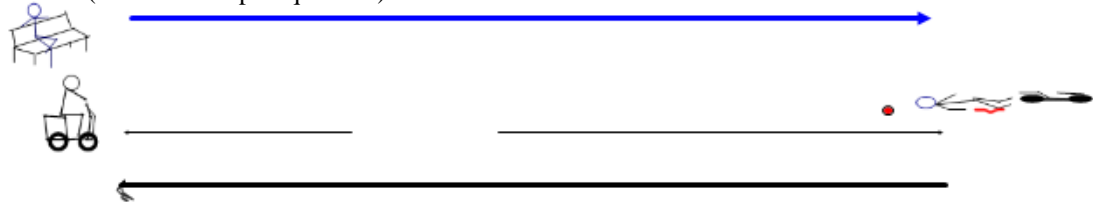


We need to look at this question as two separate parts:

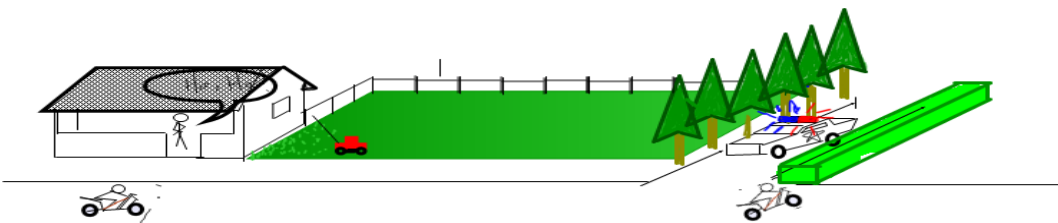
1. Time for the cocktail to get there
2. Time for the sound of the explosion to come back.

* This is not an echo.

Example W.17 Melissa is sitting at a park bench when Leon drives by on his scooter at a velocity of 54km/h . Leon travels 500m down the road when he hits an apple and wipes out. She hears him wipe out 34.79 seconds after he passed her. What is the temperature? (This is a two part question)



Example W.18 Ivan is standing on his porch when Cletus goes flying by on his motorcycle. Ivan lets out a chuckle because he knows Constable Cane is parked 700m down the road behind a large, well manicured cedar hedge. (Okay, 6 pine trees, they were easier to draw) The temperature is a comfortable 20°C . Ivan hears the siren 19.54 seconds after the bike goes by. How fast was Cletus going?

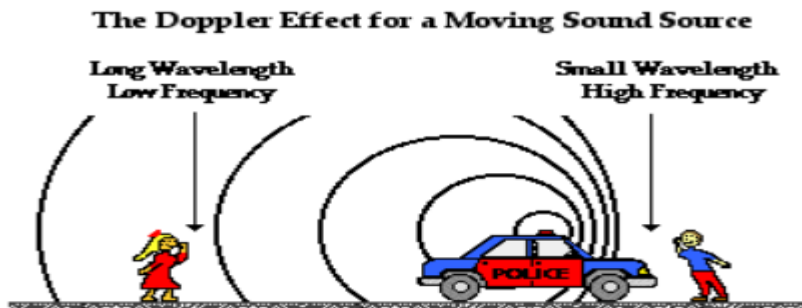
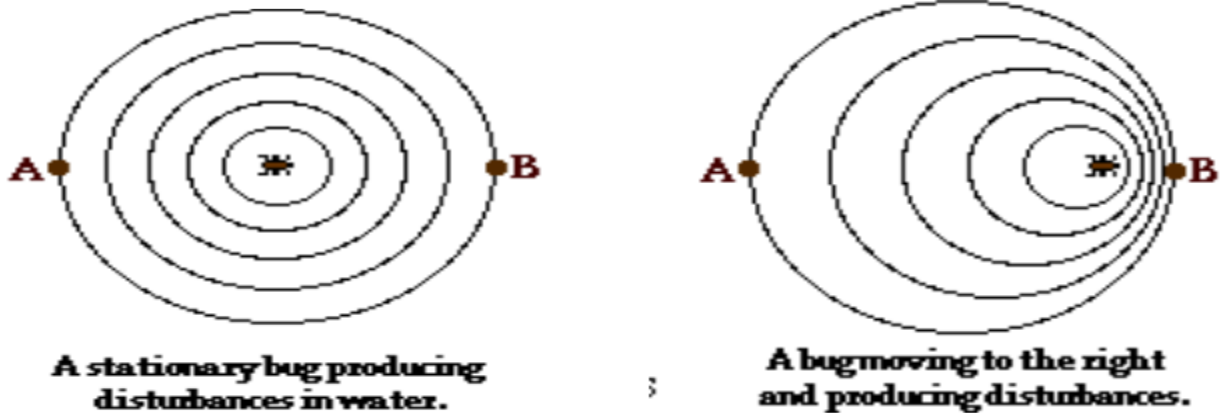


Doppler Effect

It is the apparent change in pitch when a frequency is moving. For example when a fire truck is approaching with its siren on it seems like the frequency is increasing. The pitch seems to decrease when the source of the sound is moving away.

It is affected by both the velocity of the source of the frequency and the velocity of the observer.

You can see from the diagram that as the source moves toward the observer the waves are closer together which causes an increase in frequency. The opposite is true if the source is moving away the waves are farther apart creating a decrease in frequency.



Doppler Equations

Source Moving Toward Observer

$$f' = f \frac{(v + v_o)}{(v - v_s)}$$

Source Moving Away From Observer

$$f' = f \frac{(v - v_o)}{(v + v_s)}$$

f' = perceived frequency (Hz)
 f = actual frequency of source (Hz)
 v = velocity of sound (m/s)
 v_o = velocity of observer (m/s)
 v_s = velocity of source (m/s)

Example W.19 Evelyn is watering her flowers in her front yard when she hears a siren that sounds like it is moving away from her. The siren has a frequency of 500Hz and is on a truck travelling at 30m/s. What frequency does she hear if the temperature is 10°C?

Example W.20 Louis is a volunteer firefighter. He is running towards a building at 5m/s. There is a fire alarm going off on the building. The temperature is 30⁰C and he hears a frequency of 650Hz. What is the frequency of the fire alarm?

Example W.21 Here is the scenario - Jesse is running east at 4m/s and a car is travelling west at 72km/h. The temperature is 0⁰C. The car honks its horn - frequency is 192Hz.

- a) What frequency does Jesse hear if they haven't met yet?
- b) What frequency does Jesse hear if they have already passed each other?

Example W.22 Leona is running from the police and hears a police siren from another, nearby parked cop car. The frequency of the siren is 400Hz and Leona hears a frequency of 392Hz. How fast is she running if the temperature is 10⁰C.

Example W.23 Here is the situation: Celeste is standing at an intersection waiting for the light to change when she hears a loud scream from a moving car. She detects a frequency of 560 Hz while the actual frequency is 600Hz. How fast is the car moving if the temperature is 95⁰F?

Speed of Sound, Sonic Booms and the Mach Number (Good Stuff)

The speed of sound at 20°C is 343m/s or 1234.8km/h (740.88mph). When an object moves at a speed less than the speed of sound it is moving at subsonic speeds. An object that is moving at the speed of sound is said to be moving at sonic speed or the speed of **Mach 1**. Speeds faster than the speed of sound are called supersonic speeds. Since the Mach number is the speed of sound, it is dependent on the temperature also.

When objects move faster than the speed of sound, the sound wave is moving slower than the object which creates a low pressure wave behind the object. When the high pressure wave meets the low pressure wave the result is a loud bang called a **Sonic Boom**.

Jets flying at supersonic speeds have a "sonic boom trail" behind them in the shape of a cone (think of a 3D wave behind a boat). This trail is an area of large pressure. When this wave hits the ground it may break things on the ground. As a result jets are required to fly at higher altitudes to reduce the force on the ground even though the affected area is larger.



Sonic Boom Explanation

* Scientists use a certain model when making calculations for the Mach numbers. They use what they call a standard atmosphere and use 15°C as a standard temperature. Though the temperature will change the value of Mach 1 they use this number for calculation purposes.

Mach 2 and Mach 3 are simply multiples of Mach 1

Mach 1 = 340 m/s

Mach 2 = 680 m/s

Mach 3 = 1020 m/s

Resonance Frequencies for Fixed Length Columns

* **Resonance** - (alternate definition) is a reinforcing or prolonging of sound by reflection of a wave or a vibration of other objects

This is the principle by which some wind instruments work. The player blows on the mouthpiece and the vibration of their lips creates a frequency. The length of the column determines the frequency. The sound is amplified by creating a standing wave. The pitch(frequency) of the instrument is changed by increasing/decreasing the length of the air column or changing the tension on the player's lips.

Resonance in Tubes

1. Resonance in a Closed Tube

Sounds will resonate in a tube that is closed at one end if the antinode of the sound wave is at the opening of the tube.

If you could see the actual waves in the tube you would find the resonance points similar to the diagrams below

Determining Lengths in a Closed Air Column

$$L_n = \frac{(2n-1)\lambda}{4}$$

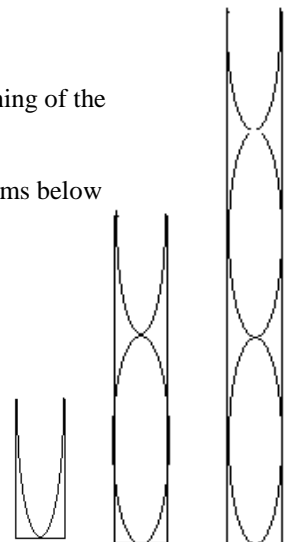
L_n = length of the tube (m)

(L_1, L_2, L_3)

n = resonance length number (1,2,3,etc)

λ = wavelength of the sound (m)

(determined by the frequency of the sound and the speed of sound)



Example W.24 The wavelength of an unknown tuning fork is 60cm. Determine the first 3 resonance lengths.

Example W.25 Using a tuning fork and a closed tube resonance apparatus you determine the second resonance point to be at 24.7cm. If the temperature in the lab is 20°C determine the frequency of the tuning fork.

Example W.26 A 680Hz tuning fork has a third resonance length at 64.2cm when used with the closed tube apparatus. What is the temperature of the room that the test is done in.

Resonance in Open Air Columns

Sounds will resonate in an air column that is open at both ends using the same principles as those for closed tubes.

Wind instruments, such as a bugle and a flute, can act as an open tube. Clarinets, oboes and bassoons work like a closed tube.

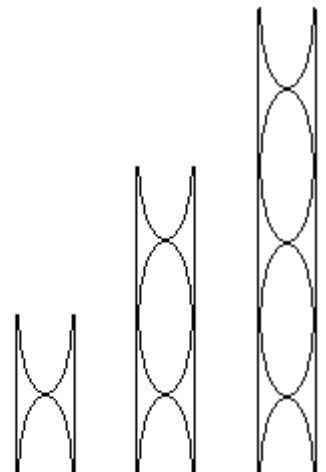
Determining the Resonance Points in an Open Air Column

$$L_n = n\frac{\lambda}{2}$$

L_n = resonance length (m)

n = resonance number

λ = wavelength (m)



Example W.27 The temperature in the lab is 20°C and you (Rodrigo) are using a candy cane colored (red, white and green) open tube apparatus and a 512Hz tuning fork. Where would you expect to find the first 2 resonance points?

Example W.28 Using the info in the previous example determine the first resonance point if you were using a closed air column.

Example W.29 Yves finds the sixth resonance point at 1.24m in a closed tube. What is the third resonance length in an open tube and what is the frequency if the temperature is 0°C.

Sound Interference

When two pure frequencies that are close, but not identical, are sounding at the same time you will hear what is referred to as a beat.

A beat is when the intensity of the sound goes from loud to quiet to loud. The loud to quiet to loud pattern is one cycle.

beat frequency - number of beats or cycles per second

Determining Beat Frequency

$$f_{\text{beat}} = | f_2 - f_1 |$$

f_{beat} = beat frequency (Hz)

f_1 = first frequency (Hz)

f_2 = second frequency (Hz)

| | = absolute value signs

$$f_{\text{beat}} = \frac{N}{t}$$

f_{beat} = beat frequency (Hz)

N = number of cycles or beats

t = time interval (s)

Example W.30 A tuning fork with a frequency of 288Hz is sounded at the same time as a string on a guitar that has a frequency of 282Hz. a) Determine the beat frequency. b) Determine the number of beats heard in 10 seconds.

Sometimes we will only be given the beat frequency but because of the absolute value signs we don't know if the difference between the two frequencies is positive or negative. To determine an unknown frequency when you know the beat frequency and one of the two frequencies we **need** to do the following:

1. Replace the absolute value signs with brackets
2. You need to do two solutions
 - i. Case 1 - put a positive (+) in front of the brackets and solve for the unknown frequency
 - ii. Case 2 - put a negative (-) and solve for the unknown frequency

* You will use this method in math, if you aren't taking a math you will only use it here!

*On a test you need to show this work to get full value for the question.

Example W.31 Pablo, the local piano tuner, hears a beat frequency of 8Hz when tuning a piano. If he is using a 512Hz tuning fork determine the possible frequencies of the piano.

Example W.32 Pablo, from the previous example, tightens the piano string and now hears a beat frequency of 4Hz. What is the current frequency and what was the previous frequency?

Example W.33 Shannon is trying to tune his guitar. He is trying to find a frequency of 440Hz. When he picks the string he hears a beat frequency of 7Hz. He then loosens the string and hears a beat frequency of 9Hz. Determine the before and after frequencies. Remember that you must show the 2 Case method to receive full marks on a test.

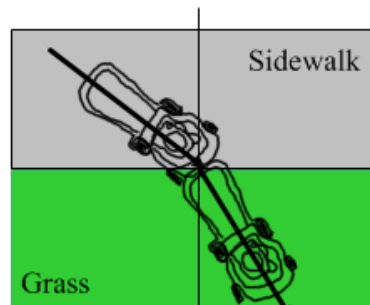
Light Waves and Refraction

Light is a wave. It travels at a velocity of $2.998 \times 10^8 \text{ m/s}$ in a vacuum. We will use $3.0 \times 10^8 \text{ m/s}$ for the sake of simplicity. Since it can travel in a vacuum it does not need a medium. However, it will still change speed when it moves into different mediums.

Refraction - changing of speed of a wave when it moves from one medium to another.

- since light is a wave it refracts
- when light goes from one medium at an angle it changes speeds but it will also change angles (this is why a pencil appears to bend when it is submerged in water)

This is an example of refraction similar to how waves bend when they go from one medium to another which we will discuss below.

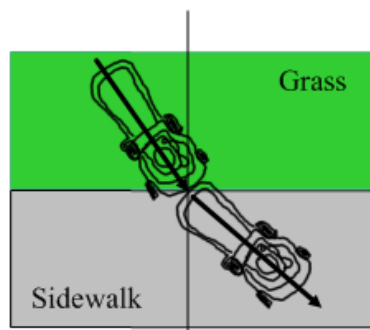


Index of Refraction

- the index of refraction is a ratio of the speed of light in a vacuum to the speed of light in a specific medium

$$n = \frac{c}{v}$$

c = speed of light in a vacuum ($3.0 \times 10^8 \text{ m/s}$)
 v = speed of light in a medium (m/s)
 n = index of refraction (no units)



The following tables are some examples of indices of refraction of some common mediums

Gases

Hydrogen	1.00014
Oxygen	1.00027
Carbon Dioxide	1.00045
Air	1.00029

Liquids at 20°C

Water	1.333
Ethyl Alcohol	1.362
Glycerin	1.470
Carbon Disulfide	1.632

Solids at 20°C

Ice	1.310
Quartz	1.460
Plexiglass	1.510
Crown Glass	1.520
Crystal Glass	1.540
Flint Glass	1.650
Zircon	1.920
Diamond	2.410

Example W.34 What is the velocity of light in glycerin?

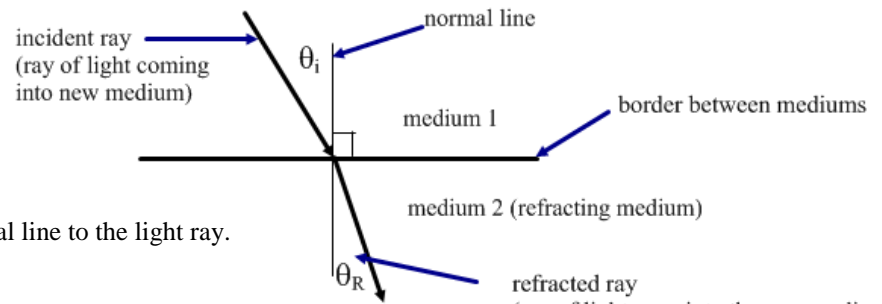
Example W.35 Helmut, with his stopwatch, determined that the velocity of light in an unknown medium was exactly $2.256 \times 10^8 \text{ m/s}$. What is the unknown medium?

***Note;** The greater the n-value, the slower the light will travel through the medium.

Snell's Observation

Willebrord Snell discovered that the ratio of sin of the angle of incidence to the sine of the angle of refraction is a constant and is equal to the index of refraction of the refracting medium. (The formula below is this statement put into words and the diagram explains the terms.)

$$\frac{\sin \theta_i}{\sin \theta_R} = n$$



*The angles are always measured from the normal line to the light ray.

After some investigation it turned out the above formula only worked if the incident medium was air. So the formula was modified to give the following, which still works with the diagram.

This is called Snell's Law.

$$n_i \sin \theta_i = n_R \sin \theta_R$$

n_i = index of refraction for first medium
 θ_i = angle of incidence
 n_R = index of refraction of second (or refracted) medium
 θ_R = angle of refraction

Example W.36 A ray of light starts in water and travels into crown glass. If the angle of incidence is 50° , what is the angle of refraction? (Include a sketch, always.)

NOTE: When you take the sine(of an angle) the answer must go to 4 decimal places (actually any trig function) For each question you must draw a sketch of what is going on. The sketch should be accurate in that the angle of incidence and the angle of refraction are scaled properly. By this I mean that if the angle of incidence is greater than the angle of refraction, then it should look like it. So how do you know which way to do it.

1. Do the question then sketch it. Of course if you made a mistake you have two wrong answers
2. Use the following rules based on index of refraction
 - the greater the n-value, the greater the index of refraction

a) if light goes from a less dense medium to a more dense medium the refracted ray bends down toward the normal (the angle of refraction is decreased) (see example above and diagrams below)

b) if the light goes from a more dense medium to a less dense medium the refracted ray bends up away from the normal (the angle of refraction increases) (see next example and diagrams below)

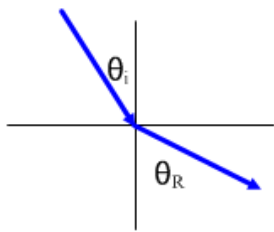
Example W.37 A drop of golden sunshine goes from zircon into an unknown medium. If the angle of incidence is 30° and the angle of refraction is 44.82° , what is the unknown medium? (Don't forget to include your sketch)

Example W.38 A beautiful beam of sunlight goes from air into water and then into crown glass. The angle of incidence is 60° . What is the angle of refraction when it is in the crown glass. (Don't forget to include your sketch) (This is a two part question in which the angle of refraction in the water becomes the angle of incidence going into the crown glass)

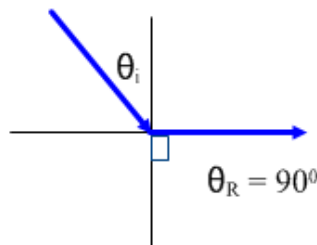
Critical Angle

It is the greatest angle of incidence that causes the angle of refraction to be 90° . It only happens when light goes from a dense medium to a light medium. When you reach the critical angle the light travels along the surface. After you exceed the critical angle the light reflects back upward at an angle that is equal to the angle coming in.

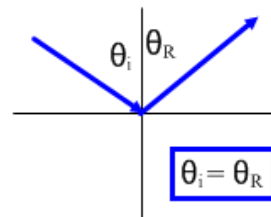
Normal Refraction



Critical Angle



Reflection



***Note :** You have exceeded the critical angle if you are trying to find the \sin^{-1} of a number greater than 1.0000.

At the critical angle the θ_R is always equal to 90° .

Example W.39 Determine the critical angle of glycerin when light travels from glycerin to water.

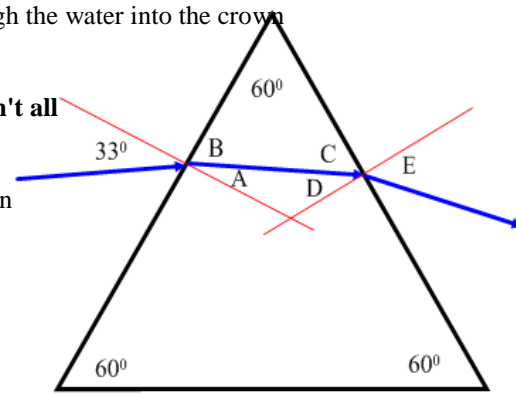
Example W.40 Light travels from crystal glass to ethyl alcohol at an angle of 75° .

- Determine the angle of refraction.
- Determine the critical angle.

Example W.41 Light travels from water to crown glass and back to water. The light comes out of the glass into the water with an angle of refraction of 35° . What is the angle of incidence coming through the water into the crown glass? (first angle of incidence)

Situations where light travels to from one medium and back but the surfaces aren't all Parallel

Example W.42 Light travels from air into a plexiglass prism and back to air as shown below. Determine the unknown angles.

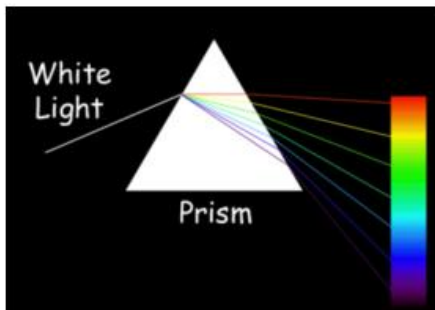


Dispersion : Rainbows and Prisms

Dispersion is defined to be the spreading out of white light into its full spectrum of wavelengths.

- we usually see it most often in light but can occur in any wave.
- we know that every medium has an index of refraction (n) but for any medium, n also depends on the wavelength.

For a given medium, **n** increases as wavelength decreases and is greatest for violet light - therefore violet light is bent more than red light as shown below. Since each color of light bends at a different rate it separates white light into the colors in a rainbow.



Wavelength (nm)	Refractive index		Colour
	Real part	Imaginary part	
400	1.34451	2.11E-10	Violet
425	1.34235	1.62E-10	Blue-violet
450	1.34055	3.30E-10	Blue
475	1.33903	4.31E-10	Cyan
500	1.33772	8.12E-10	Green
525	1.33659	1.74E-09	Yellow-green
550	1.33560	2.47E-09	Yellow
575	1.33472	3.53E-09	Orange
600	1.33393	1.06E-08	Red-orange
625	1.33322	1.41E-08	Red
650	1.33257	1.76E-08	Dark red
675	1.33197	2.41E-08	Red
700	1.33141	3.48E-08	Red

Reflection of Light

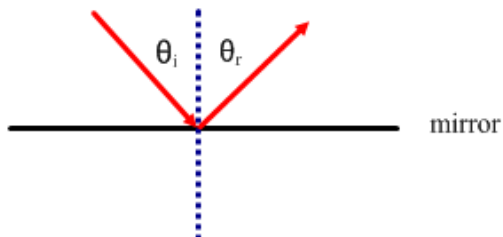
Reflection is the turning back of light waves (rays) from a surface. The **law of reflection** states that the angle of reflection is equal to the angle of incidence.

The Law of Reflection

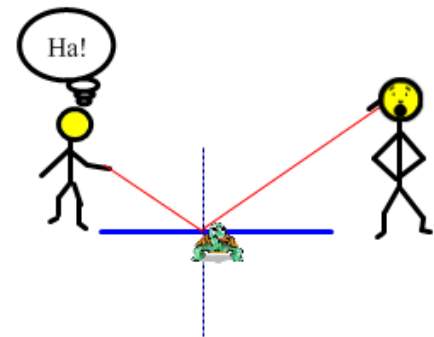
$$\theta_i = \theta_r$$

θ_i = angle of incidence

θ_r = angle of reflection

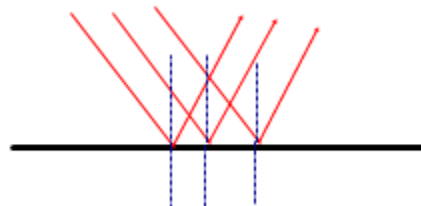


Example W.43 Billy sees his older brother Bobby watching a turtle swimming in pond 2 meters from Bobby. Bobby is 1.5m tall. Billy wants to get his brother back for a wedgie he gave him 2 hours prior. So Billy shoots his brother in the eye with his laser pointer (do not do this in real life, it could blind someone) by shining it directly where the turtle is. What angle with the normal would Billy have to aim the laser in order to hit his brother in the eye?



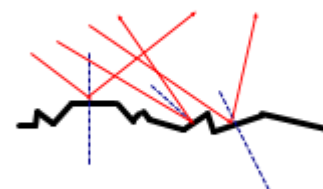
The 2 Types of Reflection

Regular reflection- the rays of light are close to parallel when reflected from a mirror. Smooth surfaces produce regular reflection, to an observer it would appear as a glare.



Diffuse reflection - the light rays are reflected in different directions because they are being reflected off an irregular surface. The law of reflection still holds, the normals are just in different directions.

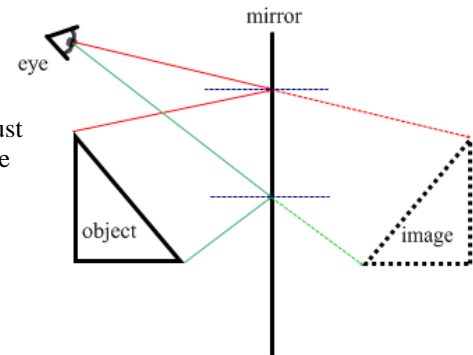
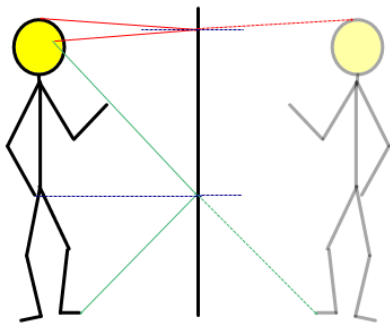
Diffuse light is important, because the human eye is sensitive to glare. This is why we have frosted light bulbs and lamp shades; these are to diffuse the light. Also if there were no particles in our atmosphere to diffuse light from the sun, shady places like under a tree would be pitch black.



Flat (Plane) Mirror reflection

The image reflected from a plane mirror is always erect (not inverted). The image is always the same size as the object being reflected. The image also appears to be as far behind the mirror as the object is in front.

Example W.44 Hank is looking at himself in the mirror. He is 1.8 m tall. How tall must the mirror be in order for Hank to see his whole body? Assume the original mirror is the same height as the person's body. The height from the top of his head to his eye is 10cm.



Curved Mirrors

For this course when we talk about curved mirrors, we are talking about a mirror made from the surface of a sphere unless otherwise specified. This is then represented on paper as an arc of a circle.

Definitions:

Concave mirror: A concave mirror is curved away from the observer.

Convex mirror: A convex mirror is curved towards the observer.

Focus: A point at which light rays meet (converge), or from which rays of light diverge.

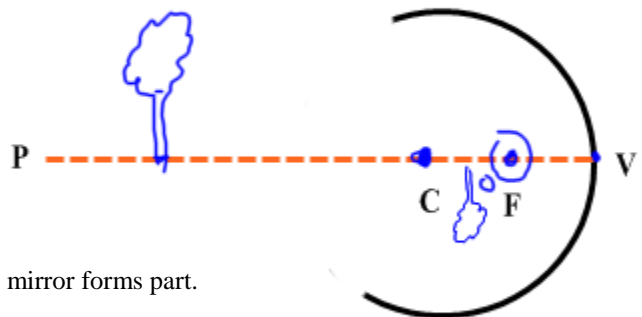
Concave mirrors

C = Center of Curvature

F = Principal Focus

V = Vertex

PV = Principal axis



Center of Curvature: is the center of the sphere of which the mirror forms part.

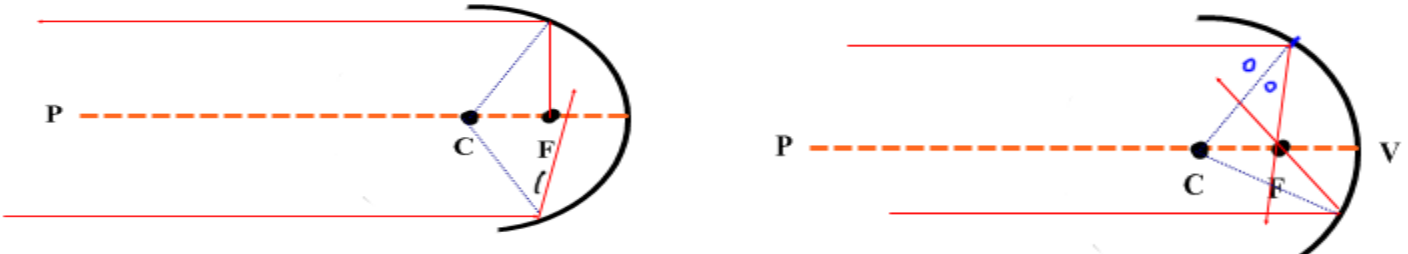
Vertex- is the center of the mirror itself.

Principal axis- is the line PV drawn through the Center of Curvature to the Vertex.

Principal focus- is half the distance between the Center of Curvature and the Vertex.

Parallel Rays Striking a Concave Mirror

When parallel rays strike a concave mirror, the rays are directed towards the principal axis of the mirror. This can be seen below, notice that the normals are the lines connecting the points of contact of the rays with the mirror to the Center of Curvature. There is a problem though, only a small section of the spherical concave mirror will direct light to the principal focus this is called **Spherical aberration**.



****Rays that make an angle of incidence greater than 45° will not be reflected to the principal focus!****

Parallel rays Striking a Parabolic Concave mirrors

Parabolic concave mirrors direct all oncoming rays towards a focus even at the extremities of the mirror. This is why they shape car headlights this way!



The 2 types of Images

Real Image- Reflected rays of light that are reflected off of a concave mirror converge at a focal point of the mirror, these rays can be projected onto a screen to form an image. This image is **real**. These images are inverted.

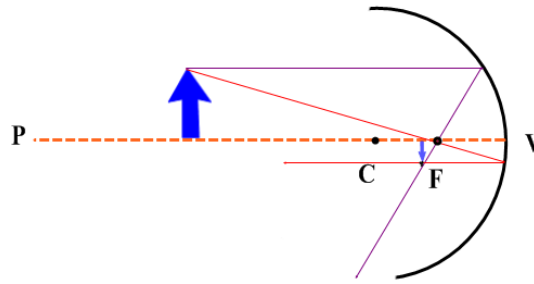
Virtual Image- The rays of light do not converge, therefore they can not be projected onto a screen, the image is called virtual. These images are never inverted, and the rays appear to originate from behind the mirror. Virtual images are what we see in a plane (flat) mirror and sometimes on convex mirrors.

Constructing an image on a concave spherical mirror

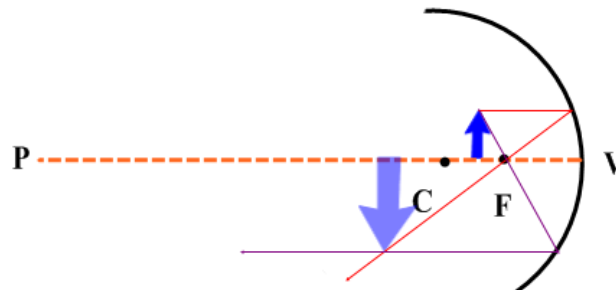
To construct an image from a concave spherical mirror, you must follow these steps:

1. Draw the mirror, the center of curvature, the focus, and your object relative to the principal focus. Make sure to draw the object on top of the principal axis.
2. From the top point of the object, draw a ray parallel with the principal axis, and reflect it back through the focus.
3. Draw a second ray straight through the focus and reflect it back.
4. The Point where the two lines meet would represent the top of the image of the object.
- 4'. If the rays diverge upon reflection, extend the lines behind the lense.
5. Where the reflected lines, and the lines drawn through the center of curvature meet is where the image of the object will be drawn.

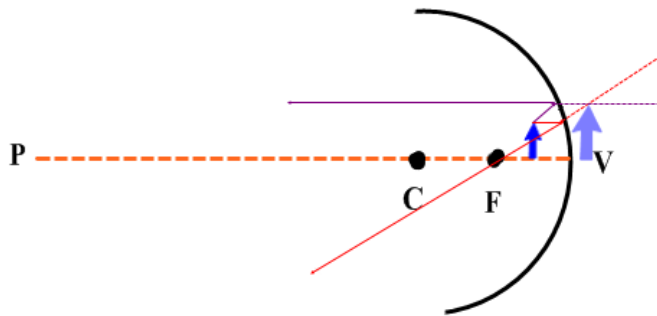
Object behind Center of Curvature



Object between Center of Curvature and Principal focus



Object Behind the Focus



The Mirror Equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

f = The focal length from the mirror
 d_i = The distance of the image from the mirror
 d_o = The distance from the object to the mirror

The above equation is used to find where the image will be with respect to the mirror.

Example W.45 If an object is 5cm from a concave mirror with a focal length of 2cm, how far away will the reflected image appear. Include a neat sketch of your work.

Example W.46 If an object is 8 cm from a mirror and the reflected image appears to be 10 cm away from the mirror, where is the mirror's focal point? Where is the center of curvature? Include a neat sketch.

Example W.47 A small garden gnome is placed in front of a concave mirror that has a center of curvature of 18cm. The gnome is placed 3cm in front of the mirror. Where would you find the image? Include a neat sketch.

Magnification Equation

The ratio of the height of the object compared to the height of the image is called the mirror's magnification. The magnification is related to the distance the object is from the mirror. If the answer is negative, it means that the image is inverted.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

m = Magnification (no units)
 h_i = Height of image (m or cm)
 h_o = Height of Object (m or cm)
 d_i = Distance of image from mirror (m or cm)
 d_o = Distance of Object from mirror (m or cm)

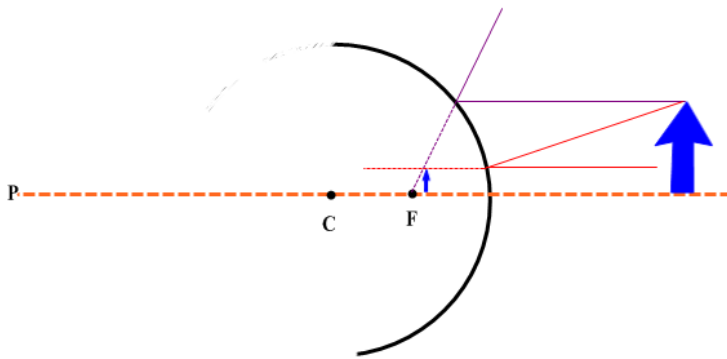
Example W.48 a) If the image and the object are on the same side of the mirror is the image upright or inverted? b) If an object is 5cm tall, and its reflection in a mirror is 15cm tall, what is the image's magnification. c) If the image is 9cm from the mirror where is the object in relation to the mirror?

Example W.49 If an object is 10cm away from the mirror and its image appears to be 15cm behind the mirror, what is the focal point of the mirror and what is the object's magnification.

Example W.50 An object 2cm high is 30cm from a concave mirror. The center of curvature of the mirror is 20 cm. What is the location of the image? And what is the size of the image?

Virtual Images Formed by Convex Mirrors

Convex mirrors always produce virtual images. The only difference between making the diagram with the convex mirror from the concave mirror is that the convex mirror the rays diverge away from the focus instead of converge towards the focus.



Example W.51 Calculate the position of an image in a convex mirror that is positioned 15cm away from a mirror with a focal length of 10cm. (Remember since f is behind the mirror it will be negative). Include a neat sketch.

Example W.52 An object is 25cm away from a convex mirror, with a focal point of 5cm. What would the distance to the image be? What would be the size of the image if the object was 8cm tall? Include a neat sketch.