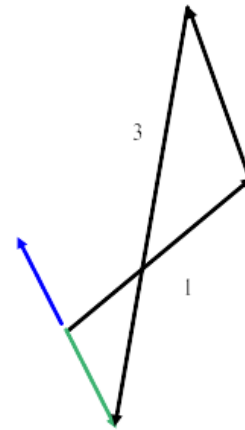
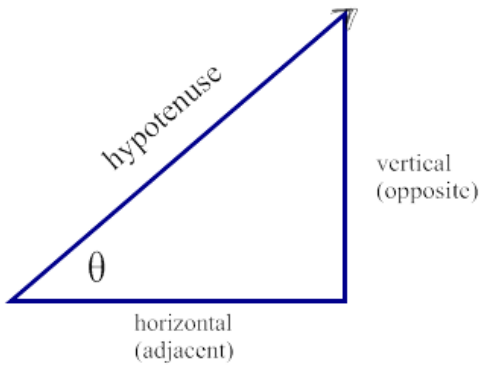
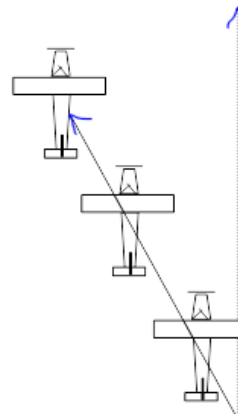
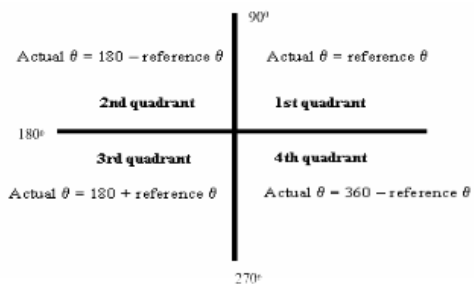


Physics 12

Course Notes



Vectors



Physics 12

Topics

- Vectors
- Fields
- Current Electricity
- Dynamics and Statics (Forces w/ Angles)
- Momentum, Projectiles and Rotational Motion

To be successful in this course you need to have a solid understanding of

- scientific notation
- vectors/basic trig (we will be using radians later)
- the need to work out the problems yourself
- the need to do all the assignments

Review

1. Scientific Notation

All answers must be written in proper scientific notation.

Ex. 3.478×10^8 (correct) 347.8×10^6 (Incorrect)

Changing to proper notation

- there should be one number (1-9) followed by a decimal
- if you move the decimal place to the right (make the number smaller) you need to add to the exponent (make it bigger)
- the opposite is true when moving the decimal to the left

Example V.1: Change the following to proper notation

1. $4693.23 \times 10^6 = 4.69323 \times 10^9$

2. $927.6 \times 10^{-17} = 9.276 \times 10^{-15}$

3. $0.00834 \times 10^8 = 8.34 \times 10^5$

4. $0.000386 \times 10^{-31} = 3.86 \times 10^{-35}$

Using your calculator

Remember to use the EXP or EE buttons (know where they are and how to use them)

4213.99×10^9 is 4213.99EE9 and will give you 4.213 12 which **means** 4.213×10^{12}

Multiplying/Dividing in Scientific Notation

Your calculator will do it but if multiplying then multiply the numbers and add the exponents

For dividing you divide the numbers and subtract the exponents

Examples

$$3.0 \times 10^6 \times 4.0 \times 10^8 = (3.0 \times 4.0) \times 10^{6+8} = 12.0 \times 10^{14} = 1.2 \times 10^{15}$$

$$1.50 \times 10^{-21} / 3.0 \times 10^{-5} = (1.50 / 3.0) \times 10^{-21 - (-5)} = 0.5 \times 10^{-16} = 5.0 \times 10^{-17}$$

Adding/Subtracting in Scientific Notation

- The exponents must be same, safest bet is to use your calculator

Converting Units

- you should know how to do it. Refer to your Grade 11 notes if you don't (you still have them right?)

remember km/h -> m/s 3.6
m/s -> km/h x 3.6

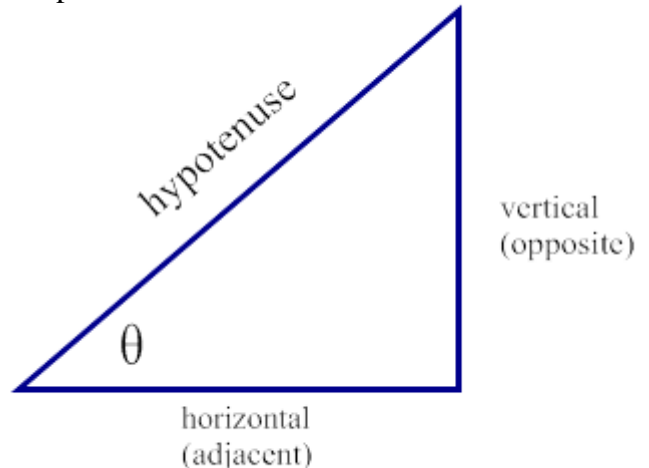
Basic Trig Functions

- We use them to determine Horizontal and Vertical Components of vectors

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{\text{vertical (y)}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{\text{horizontal (x)}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\text{vertical}}{\text{horizontal}}$$



We will rearrange the above equations to be able to solve for the horizontal and vertical components of forces, momentums, velocities, etc., at angles.

Vertical component (y - comp)

$$y = \text{hyp} \sin \theta$$

Horizontal component (x-comp)

$$x = \text{hyp} \cos \theta$$

*Your hypotenuse is the magnitude of the vector in the question. If you are using forces it is the force. If you are using displacement it is the displacement

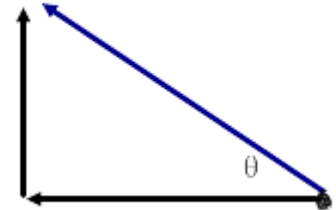
Example V.2: A force of 100N is applied at an angle of 30° . Determine the x and y components. Always include a sketch.

* Note: We will be measuring our angles from the horizontal. As such the **y-component** is always with **sine** and the **x-component** is always with **cosine**.

This concept will work for any angle and it automatically gives you the negative so that you don't have to guess. You should be able to tell whether the components are positive or negative based on the angle.

For example in this triangle the x-component would be negative and the y-component is positive.

Since we are measuring from the x-axis we ALWAYS draw the X-component first.



Vectors

vector - has magnitude and direction

- can represent displacement, velocity, force, electric field intensity, momentum, etc

We will be using vectors extensively but they are not difficult

There are two ways to solve vectors:

1. graphically
2. mathematically

We can use this method to solve when several vectors are acting simultaneously and we are trying to determine the resultant vector

resultant - is the vector that would give the same final displacement as all the vectors added together

Adding Vectors Graphically

- you need a ruler and a protractor as they must be drawn accurately
- choose a starting point and a scale (for example $10\text{N} = 1\text{cm}$)
- start by drawing the first vector (since you are adding the order doesn't matter)
- from the head of the first vector draw the second vector (each vector must be drawn with the protractor set so that the 0° is pointed directly to the east(right))
- continue until all vectors are drawn
- draw a vector from the start point to the end point. This is the resultant
- measure the length and angle of the resultant (remember to convert the length back to the proper units from your scale)

equilibrant - a vector that is equal in magnitude but in the direct opposite direction (180°)



Example V.3: Jacob goes for a walk. He starts out heading east for 30m. He then heads north for 40m and finally he turns and walks at 135° for 60m. Determine his resultant displacement.

Example V.4: Three people are pulling on the same item at the same time. Jack is pulling with a force of 400N at 40° . Angela is pulling with 300N at 110° . Chris is pulling with 700N of force at 260° . Determine the resultant force and the equilibrant force. Remember to include the directions.

Example V.5: Determine the resultant when the following 4 forces act simultaneously.

1- 6000N at 10°

2 - 4500N at 80°

3 - 7000N at 200°

4 - 2500N at 110°

Adding Vectors Mathematically

–Create a table that has a column for the vector, and two other columns - one for the y-component of the vector and one for the x-component of the vector

–Break each vector into its x and y components by using sine and cosine (remember x - cos, y - sin)

- Add all the x-components together

- Add all the y-components together

- Sketch a triangle with the total(net) x and y components as the sides and determine the hypotenuse

and the angle between the **horizontal** (using tangent) **Remember to include negatives here.**

–This gives the magnitude and reference angle of the resultant (You must draw the triangle)

- determine the actual angle of the resultant using the following chart

Note : draw x first, then draw y second and then close in the triangle with the resultant (hypotenuse)

Example V.6: Jacob travels 30m at 0°, 40 m at 90° then 60 at 135°

Solution:

Vector	Y-component	X-component
30m at 0	30sin 0 = 0	30cos 0 = 30.00
40m at 90	40sin 90 = 40.00	40cos 90 = 0.00
60m at 135	60sin 135 = 42.43	60cos 135 = -42.43
Net	82.43	-12.43

Draw the resultant triangle by drawing the net sum of the **x-components first** and then the net of the **y-components second**. Complete the triangle by drawing the hypotenuse (a.k.a the resultant) **ALWAYS DRAW THE TRIANGLE**

Find the length using Pythagoras Theorem

$$R^2 = x^2 + y^2$$

$$R^2 = (-12.43)^2 + (82.43)^2$$

$$R^2 = 6949.21$$

$$R = 83.36$$

$$\tan \theta = \frac{y}{x}$$

$$x$$

$$\tan \theta = \frac{82.43}{12.43}$$

$$12.43$$

$$\tan \theta = 6.6315$$

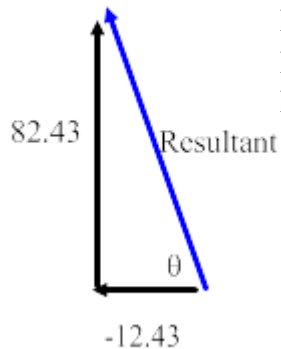
$$\theta = \tan^{-1}(6.6315)$$

$$\theta = 81.42^\circ$$

$$\text{Actual } \theta = 180 - \text{ref } \theta$$

$$\text{Actual } \theta = 180 - 81.42$$

$$\text{Actual } \theta = 98.58^\circ$$



The resultant is 83.36m at 98.58°

Note : The angle we have found is the reference angle in the triangle but it is not the angle that the vector is pointing at.

Finding the actual angle of the resultant

- Sketch the triangle
- Determine the quadrant (this is why we draw the x and then the y)
- Determine the reference angle using tangent and ignoring the negatives
- Use the reference angle and the chart below to determine the actual angle

Actual $\theta = 180 - \text{reference } \theta$	Actual $\theta = \text{reference } \theta$
2nd quadrant	1st quadrant
3rd quadrant	4th quadrant
Actual $\theta = 180 + \text{reference } \theta$	Actual $\theta = 360 - \text{reference } \theta$

Example V.7: Thomas applied a force of 60N @ 10° to a mystery object. Larry applies a force of 45N @ 80° at the same time. Rhonda joins the party and applies a force of 70N @ 200° . A fourth masked person (witnesses described someone who looked a lot like Batman) applies a force of 25N @ 110° . Determine the resultant force and angle.

Vectors with Scalars

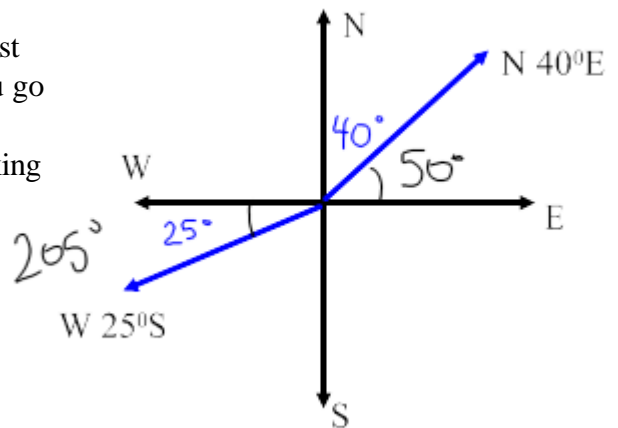
Sometimes we use a vector, such as velocity, with a scalar, such as time, to find a different value - in this case displacement

- in order to work with multiple velocities they either a) must act on the same object simultaneously or b) must happen in two different times (drove in one direction then another) but the times must be exactly equal - OTHERWISE find displacement. See example below

Note: Remember we use $x = x_0 + \bar{v}t$ to with displacement

Compass directions

- measurements that would be used on a compass
- **angle is never more than 45°**
- always stated by the compass heading that is the closest
- for example W 25° S means it is looking west then you go 25 degrees toward the south direction
- another example might be N 40° E - which means looking north then you turn 40 degrees to the east



Example V8 : A hot-air balloon racer is in the air and drifts with the wind at a rate (velocity) of 24 km/h [E40°N] for 2 hours. The wind shifts, causing the racer and his balloon to drift south at a rate of 40 km/h for 2.5h. a) Determine the balloon's displacement for the flight (Express in compass direction) b) What is the racer's average velocity for the trip [Express in compass direction]. NOTE: We have to convert to displacement first since the times are different.

Finding a Missing Vector if you Know the Resultant

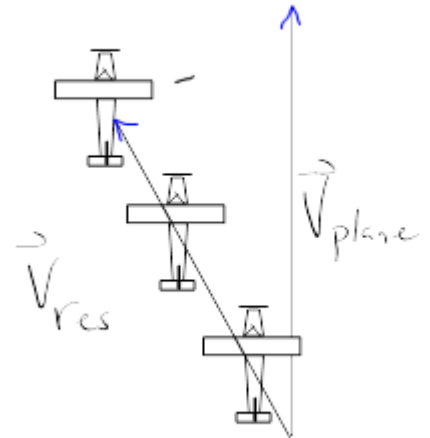
Example V.9: Three friends have decided to hook their trucks together and see where they will end. Determine the force that the third truck applies if the resulting force and direction is 4172.12N @ 246.38. The following is the force each truck can apply and the angle the force is applied at:

Truck 1: 8000N at 45°

Truck 2: 9000N at 200°

Example V.10: A small airplane is heading pointing due north and travels with a velocity of 45.0 m/s. However, due to the wind the plane ends up with a velocity relative to the ground of 38.0m/s at direction of N20°W. What direction is the wind blowing?

The plane is pointing toward the north but the wind is continually pushing it such that it looks, from the ground, like it is going at the direction that is stated. This is the resultant direction of it's velocity and the wind pushing on it. Therefore we are looking for a missing vector not the resultant.



Resultant and Equilibrant

- resultant is the result of all the forces applied, the displacements travelled, etc.
- equilibrant is the force, displacement required to bring everything back to zero or into equilibrium
- the resultant always has an angle between 0° and 360°
- the equilibrant is the same so
 - if the resultant is less than 180° you will need to ADD 180° to find the angle
 - if the resultant is more than 180° you need to SUBTRACT 180°

Vectors, Cosine Law and Sine Law

- if you have two vectors you can use the Cosine Law to find the resultant.
- you do not need to use this method unless you want to or you are doing Level 1

$$\text{Cosine Law}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Sine Law}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example V.11: Two forces are applied. The first is 40N at 20° and the second is 30N at 120° . Determine the resultant

You need to use the calculated inside angle of 80° .

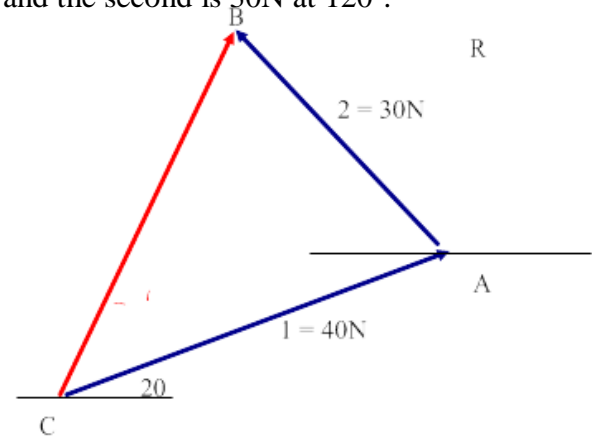
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 40^2 + 30^2 - 2(40)(30) \cos (80)$$

$$a^2 = 1600 + 900 - 416.76$$

$$a^2 = 2083.24$$

$$a = 45.64$$



Work Backwards using the cosine law or use the sine Law to find the angle. Remember that this is the inside angle so you will need to add the outside angle.

Since we want C and we have A

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{45.64}{\sin 80} = \frac{30}{\sin C}$$

$$\sin C = \frac{30 \sin 80}{45.64}$$

$$C = \sin^{-1}(0.6473)$$

$$C = 40.34^\circ$$

Angle of the resultant is $40.34 + 20 = 60.34$
The resultant is 45.64 at 60.34° .

Vector Algebra

Vector algebra is used in university physics and therefore touched on here. The concept is essentially what we have been doing but we express our answers in a different form. We can express a vector as its components

$$\vec{v} = v_x + v_y = v_x \hat{i} + v_y \hat{j}$$

To apply this concept. The resultant of the addition of two vectors is what we have been talking about so far. We could write it in basic form as

$$\vec{R} = \vec{A} + \vec{B} \quad \text{if A and B are at angles then they need to be broken into x and y}$$

$$\vec{R} = A_x + A_y + B_x + B_y \quad \text{we have rewritten it in its components}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} \quad \text{we have combined the x components just as we did when we found the sum of the x-components and the sum of the y-components}$$

Example V.12 Using example V.6 write the resultant in i, j form. *Example 6: Jacob travels 30m at 0°, 40 m at 90° then 60 at 135°*

This was the solution:

Vector	Y-component	X-component
30m at 0	30sin 0 = 0	30cos 0 = 30.00
40m at 90	40sin 90 = 40.00	40cos 90 = 0.00
60m at 135	60sin 135 = 42.43	60cos 135 = -42.43
Net	82.43	-12.43

We could write it as:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} \quad \text{where A is the first vector, B is the second vector and C is the third}$$

$$\vec{R} = A_x + A_y + B_x + B_y + C_x + C_y$$

$$\vec{R} = (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j}$$

Since we know the value we can put them in

$$\vec{R} = (30 + 0 + -42.43)\hat{i} + (0 + 40 + 42.43)\hat{j}$$

$$\vec{R} = -12.43\hat{i} + 82.43\hat{j}$$

Note these are the net values of the x and y, except this is the final answer. We don't need to find the resultant or angle. This is the end, the rest is math. In university they use this method to see if you can solve the problem without wasting time on the math part.

Vector Subtraction

Sometimes we need to subtract vectors instead of add them. A common scenario is for calculating acceleration. If an object moves in a circle at a constant speed its direction is always changing. Since acceleration is a change of speed or direction we can find acceleration even if the speed stays the same. Remember acceleration is the change in velocity divided by the time. So how do you subtract vectors? Check out the examples below.

Subtracting vectors is the same as adding except you turn one of them into a negative vector. For example if you wanted to solve the following situation it would go as follows:

$$\vec{R} = \vec{A} - \vec{B}$$

$$\vec{R} = \vec{A} + (-\vec{B})$$

This would be fairly easy mathematically. Whatever the B vector components are you just make them the opposite. If you are solving graphically you do everything the same as we did for adding but you draw the second vector in the opposite direction (180°).

Example V.13 Two vectors are as given:

A is 40m at 90° and B is 50m at 180° . Determine the resultant of $A - B$.

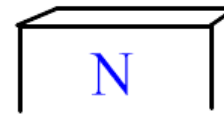
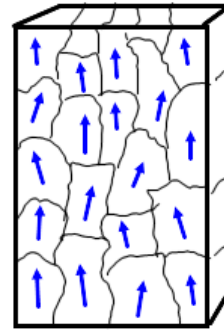
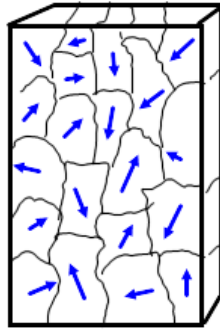
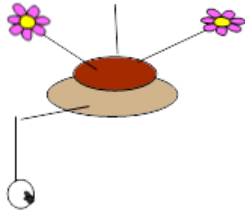
	x-comp	y-comp
A 40m @ 90°	0	40
B 50m @ 180°	-50	0
	50	40

At this point we have two choices. 1) $A - B$ or 2) $A + (-B)$

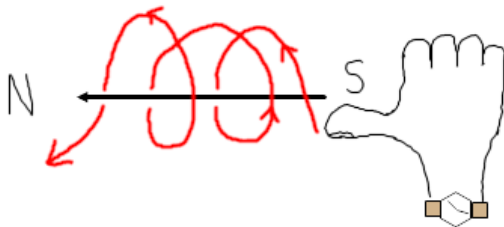
If you are subtracting put a (-) out front of vector B or if adding the negative change the angle.

If doing graphically we draw B in the opposite direction but we still add.

NOTE: If doing something like acceleration where we use velocity we doing $V_2 - V_1$. Remember acceleration is final velocity minus initial.



Fields



Fields and Forces

Fields

- We will be looking at electrostatic, magnetic and gravitational fields
- A field will exert a force of attraction or repulsion on other objects in the field
- All of these fields have forces which follow the Inverse Square Law

Inverse Square Law

The force of attraction/repulsion between two objects is inversely proportional to the square of the distance between them. The greater the distance between them the weaker the force. When we talk of proportionality we assume all other variables in the equation stay constant and we look at the effect that one particular variable has on another variable in the equation.

$$F \propto \frac{1}{r^2}$$

F = force

r = distance between the objects

\propto = Proportional (we assume all other variables stay the same)

Example F.1: Two objects are 4m apart. Find the force difference if they are moved:

- 12m apart
- 1 m apart

Brief History of Electrostatics (Static Electricity)

Charles Coulomb (real name) is credited with some of the first discoveries in electrostatics. He used some pith balls (similar to Styrofoam) and a static charge to determine the force between charged particles. Without going into the details he came up with Coulomb's Law.

Coulomb's Law

- used to determine the force between charged particles

$$F_Q = \frac{kq_1q_2}{r^2}$$

F_Q = electrostatic force (N)

k = Coulomb's constant ($9.0 \times 10^9 \text{ Nm}^2/\text{C}^2$)

q_1 = charge of first object (C)

q_2 = charge of second object (C)

r = distance from the center of a charged sphere (m)

** Since 1 Coulomb can generate a substantial force we work in μC (micro coulomb = 10^{-6}C)

*If the charges are the same then they will repel each other

*If the charges are opposite then they will attract each other

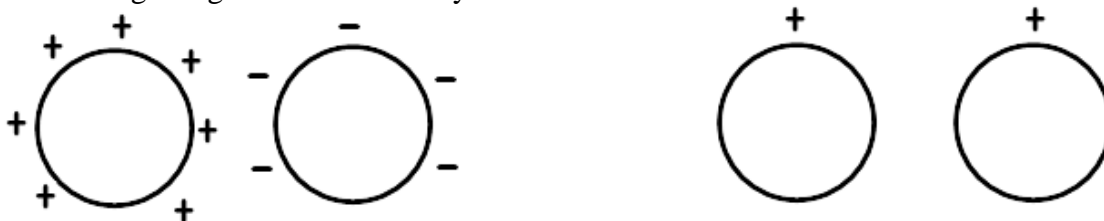
* All the charges on a sphere will spread evenly around the surface of the sphere but acts as if all its charge is concentrated at the center

Example F.2: Two charged spheres are placed near each other. The force is 5N and it is an attractive force. What is the charge of the second sphere if they are 0.85m apart and the first sphere has a charge of $12\mu\text{C}$?

Example F.3: Determine the force between the charges in the above example if they are moved such that they are 1.70m apart.

Special Situation - Two charged Spheres touched together

When two spheres with different charges are touched together they will reach equilibrium such that each sphere will have the same charge. To determine the charges after contact add the amount of charges together and divide by 2.



*Each positive is balanced by a negative and therefore balanced. In this case it will leave 2 positive charges which will split between the two spheres.

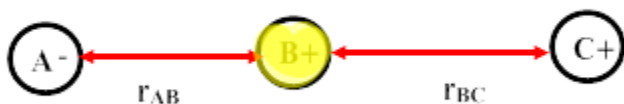
Forces with 3(or more) Charged Objects

2 Types:

1. All objects in a straight line. Find the force between pairs and determine the net force and state the direction using your knowledge of attraction/repulsion
2. Objects are not in a straight line. We need to find the force and direction between pairs and determine the net force and direction using vectors.

*Vectors can be used to solve type 1 also

Objects in a straight line



If we are trying to find the force B

- determine the force between A and B
- determine the force between B and C
- determine the net force on B (use the signs to determine attraction/repulsion and the direction of the force)

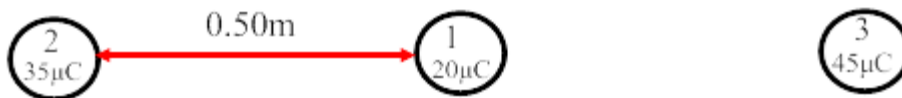
Remember : $F_{\text{net}} = \text{Sum of all forces} = \text{forces to the right} - \text{forces to the left}$

Example F.4 Three charged spheres are aligned in a horizontal line. The first sphere has a charge of $10\mu\text{C}$, the second has a charge of $-15\mu\text{C}$ and is located 25cm to the right of the first sphere. The third sphere has a charge of $-20\mu\text{C}$ and is located 60cm to the right of the first sphere. Find the net force and direction on the second sphere, include a diagram.

Forces between objects when they are in a line

The process is still the same except we are looking for or are given the net force on one of the objects. Solve for the forces and treat it like a regular F_{net} /FBD situation.

Example F.5 You are given the following scenario. You are working with 3 spheres. The first sphere has a charge of $20\mu\text{C}$ and the second has a charge of $35\mu\text{C}$ and is located 50cm to the left. If the third sphere has a charge of $45\mu\text{C}$ where would you put it so that the force on the first sphere is balanced?

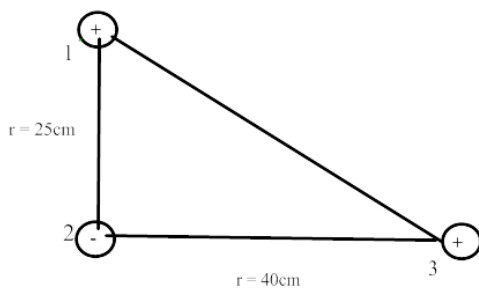
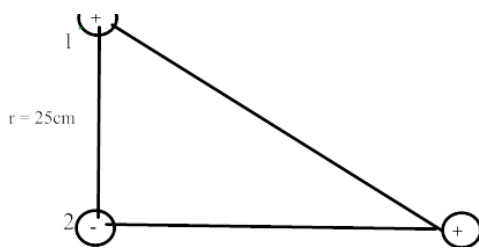
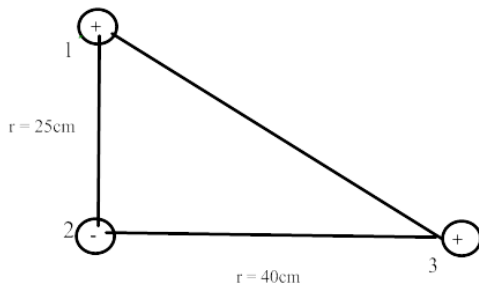


Forces Between 3 or 4 more Charges Not in Line (Random Location)

- we will be asked to determine the force on a specific object
- we still need to find the force between objects required but we need to determine the sin and cosine of the angle that each force is acting at
- sometimes we will need to determine the distance between objects
- repeat for as many pairs as necessary
- after we determine the force and direction we have to determine the resultant using vector analysis (See Vectors unit)
- always draw the force vector as it acts on the object in question. Complete a triangle by drawing a horizontal then vertical line.
- if the triangle has dimensions use them, if not extend down to the other object to complete a triangle with dimensions

Example F.6 Determine the direction of the force in each of the situations below

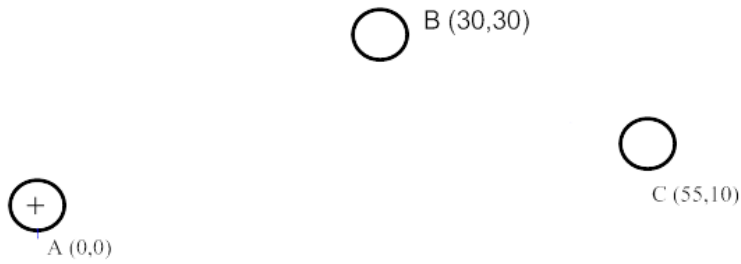
- Determine the sin and cos of the angle of the forces on object 1
- Determine the sin and cos of the angle of the forces on object 2
- Determine the sin and cos of the angle of the forces on object 3



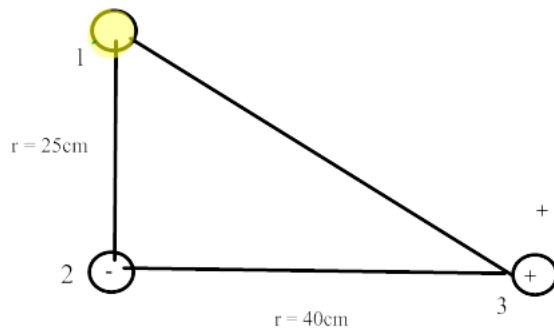
Example F.7 Three charged spheres are located as follows: Sphere A is positively charged and is located at point (0,0). Sphere B is located at (30,30) and is negatively charged. Sphere C is located at (55,10) and is positively charged. All dimensions are in cm. Draw a diagram to locate the spheres and determine

a) the sin and cos of the angle of the forces acting on sphere B

b) the sin and cos of the angle of the forces acting on sphere C



Example F.8 Using the information in example F.6 determine the resultant force on sphere 1 if sphere 1 has a charge of $+25\mu\text{C}$, sphere 2 has a charge of $-30\mu\text{C}$ and sphere 3 has a charge of $+40\mu\text{C}$.

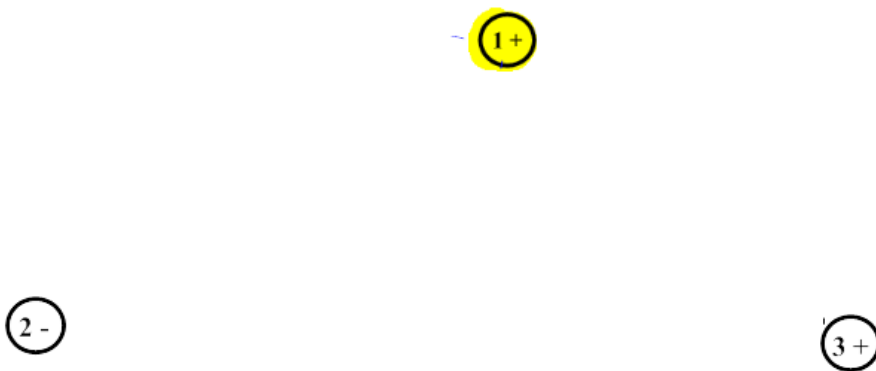


Summary

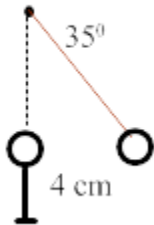
If you can follow the steps the process is the same regardless of how many objects you have. This basic process also works for the fields section we will be doing next.

1. Determine the lengths between 1-2 and 1-3 or whatever objects you are dealing with
2. Determine the forces between 1-2 and 1-3 or whatever objects you are dealing with
3. Find the sin and cos of the angles of the forces.
4. Determine the x and y components
5. Determine the magnitude and direction of the resultant
6. Make a summary statement

Example F.9 Using the following diagram determine the forces acting on object 1. Object 1 is located at (0,0) and has a charge of $50\mu\text{C}$. Object 2 is located at (-75, -40) and has a charge of $-40\mu\text{C}$. The third object has a charge of $+60\mu\text{C}$ and is located at (50, -40). All dimensions are in cm.



Example F.10 Damon has assembled an apparatus to determine the charge on a pith ball. He hangs a 3g pith ball on a string. He has another pith ball on a stand which is charged. Damon brings the ball on the stand over to the string ball and touches them together. After they touch the hanging ball moves so that it is 4cm away horizontally. The string makes an angle of 35° with the horizontal. Determine the charge on each ball.



Example F.11 If the two pith balls above were changed to a mass of 4 grams would you be able to determine the charge of the pith balls. Support your answer.

Describing Fields (Electrostatic, Gravitational and Magnetic)

Electric Field Intensity

- a charged body will generate an electrostatic field around it which may repel or attract
- to determine the intensity of the field at a certain point we place a test charge in the field and the force on the charge is determined
- the intensity varies depending on the distance from the charge

Magnetic Field Intensity

- same principles as electrostatic fields in that it may repel or attract
- we can determine the direction of the field as either south or north depending on the magnet
- intensity also varies depending on distance

Gravitational Field Intensity

- every body will generate a gravitational field around it
- the field is similar to the previous two except the direction of the field is always inward towards the center of the source

Type of Field	Attract	Repel	$1/r$	$1/r^2$
Electrostatic	√	√		√
Gravitational	√			√
Magnetic	√	√		√

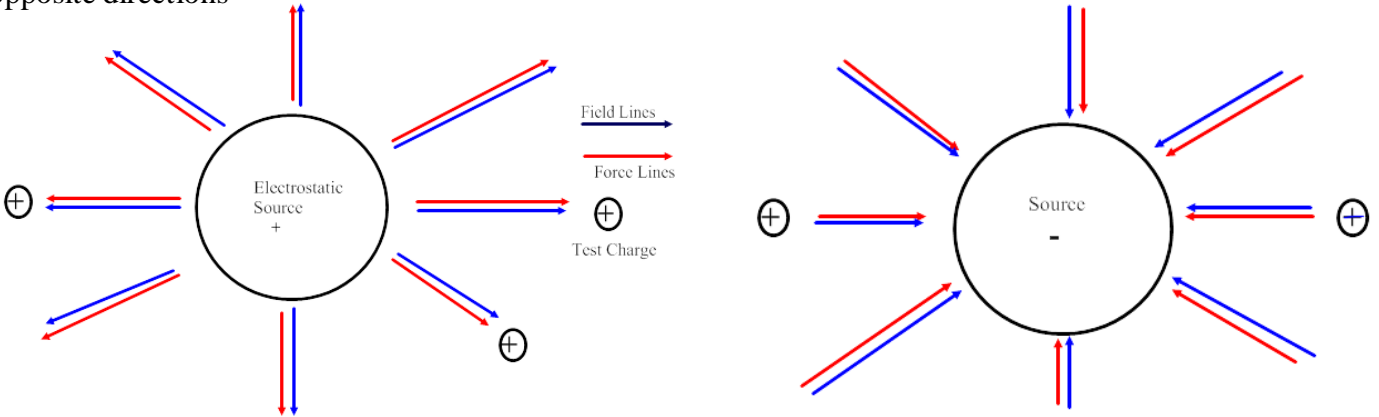
Electric Fields

Direction of Force

- in an electrostatic field the direction of the field is the **SAME** as the direction of the force that is exerted on a positive test charge
- to understand field intensity we use a test charge because we don't always know the charge of the source that generates the field or where the field is being generated from

Field Lines

- Field lines help determine whether a source will cause attraction or repulsion
- Field direction lines are constant, once they are determined they don't change regardless of any new charges (think of them as a warning sign)
- For a positive test charge the direction of the field and the direction of the force are in the same direction
- For a negative test charge the direction of the field and the direction of the force are in opposite directions



- *Important - once the field lines are determined they stay the same, regardless of the test charge.
- the direction of the force can change, we can use the direction of the force and the sign of the charge to determine the field lines also

Example F.12 Determine the direction of the field based on the following situations:

- A positive test charge is placed in an electric field and experiences a force to the right
- A negative test charge is placed in an electric field and experiences a force to the right

Determining Field Intensity When you Don't Know the Source Charge

Field Intensity is the amount of force applied for each Coulomb of charge when it is near a charged source. It can be determined by the following formula:

$$E = \frac{F_Q}{q_T}$$

E = field intensity (N/C)
F_Q = electrostatic force (N)
q_T = test charge (C)

Example F.13 A positive test charge of 3μC is placed in an electric field and experiences a force of 6 x 10⁻⁶N to the left.

- What is the intensity and direction of the field at this location?
- What force would a -6μC charge feel at the same point?

Field Intensity Near a Point Source

If we know the magnitude of charge for the source we can use the following formula:

$$E = \frac{kq_s}{r^2}$$

E = field intensity (N/C)
q_s = charge of source (C)
r = distance from the center of the source (m)
k = electrostatic constant (9x10⁹Nm²/C²)

Example F.14 a) Determine the charge of a sphere that generates an electric field if the field intensity from the source is 3.0x10⁶N/C at a point 10cm from its center. b) What force would a -0.5μC test charge feel at this location? c) If you tripled the distance away from the source what would the field intensity be?

Example F.15 A test charge of $+6.5\mu\text{C}$ is placed to the right of a source in an electrostatic field and feels a force to the left of 28N . a) Determine the charge of the source if the test charge is placed 20cm from the source. b) Determine the field intensity at this location

Behavior of Electrical Charges

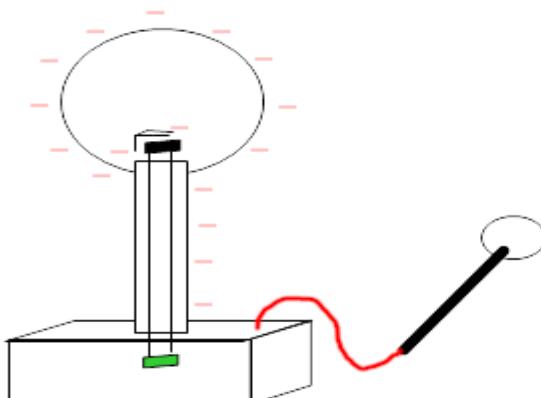
1. Why do you get a shock when you walk across the carpet and touch a door knob or when you get out of a car and touch the door?
2. How does the Vandegraaf (electrostatic) generator work?
3. How can you bend water with a rod after you rub it with a piece of fur?

An electrical charge is simply a bunch of electrons. Friction between two surfaces can cause electrons to move. (Friction is an electrostatic force.) Electrons will then move to areas of least resistance to attempt to achieve equilibrium. Whether they move easily or not is based on the type of material.

Materials can be broken into 2 groups

1. conductors - electrical charges flow easily over the outside of the material (ex. metal, non-distilled water)
2. insulators - when a charge is on an insulator it will stay in one spot (ex. wood, plastic, glass)

Vandegraaf Generator



How does it work?

Field Intensity Near Several Point Sources

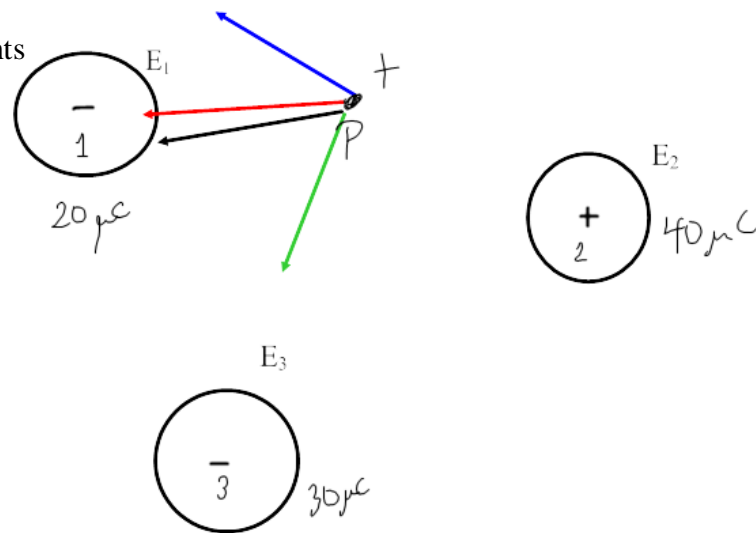
We calculate the resultant field intensity at a point in all their fields in exactly the same manner as we determined the resultant forces acting on point with the exception that we are determining the field intensity (E) for each of them. (**The point P is always assumed to be a positive test charge**)

To visualize such a scenario, imagine trying to determine where a small piece of charged paper would go if it were placed in the electrostatic fields of 2 or more Vandegraaf generators.

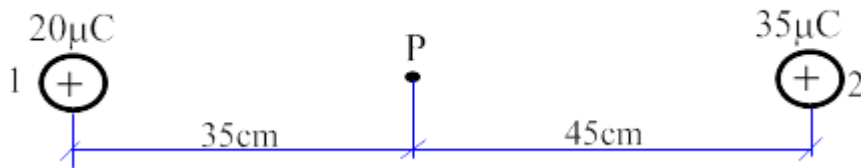
Process

1. Find the distance from the point to each source.
2. Find the angle that the point would move to if allowed to move.
3. Determine the field intensity for each source
4. Break the field intensities into x and y components
5. Solve for the resultant field intensity.

Here is a diagram to help picture a scenario:



Example F.16 Determine the electrostatic field intensity at point P in the diagram below.



Example F.17 The field intensity at point P due to the 2 different point sources in the following diagrams are given as follows:

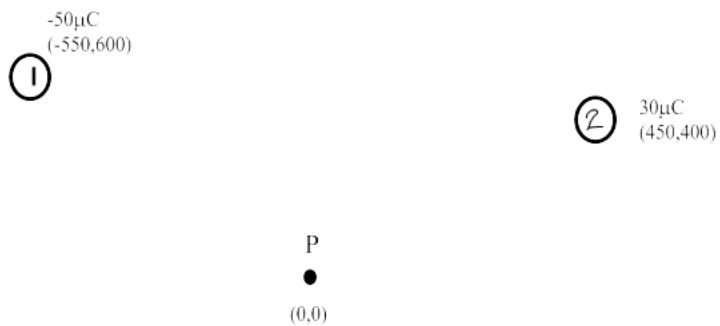
$$E_1 = 4.3 \times 10^6 \text{ N/C}$$

$$E_2 = 1.9 \times 10^6 \text{ N/C}$$

Solve for the resultant field intensity.



Example F.18 Determine the resultant field intensity at point P due to the 2 different point sources in the following diagram. All distances are given in millimeters.



Gravitational Fields

- same concept as electrostatic fields except the direction of force is always attraction toward the larger mass(aka the source)
- whereas Electrostatic field intensity is the amount of force on a charged test charge, gravitational field intensity is amount of force on a test mass.
- the formula should look familiar we are just looking at it from a different angle

Gravitational Field Intensity with a Mass

$$g = \frac{F_g}{m}$$

g = gravitational field intensity (N/kg)
 F_g = gravitational force (N)
 m = test mass (kg)

*Note: we always measure any distance from the center of the body

Example F.19 A 460kg object feels a gravitational force of 700N at a location several kilometers above the moon's surface. What is the gravitational field intensity at this point?

Gravitational Field Intensity Near a Source Mass

$$g = \frac{Gm_s}{r^2}$$

g = gravitational field intensity (N/kg)
 m_s = mass of the source (kg)
 r = distance from the center of the source (m)
*for a planet ($r = r_{\text{planet}} + h$) where h is height above surface
 G = universal gravitational constant ($6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$)

Important values (You may want to add more as they show up in examples or questions)

Masses

$m_{\text{earth}} = 5.98 \times 10^{24} \text{ kg}$
 $m_{\text{venus}} = 4.83 \times 10^{24} \text{ kg}$
 $m_{\text{saturn}} = 5.67 \times 10^{26} \text{ kg}$
 $m_{\text{mars}} = 6.42 \times 10^{23} \text{ kg}$

Radii

$r_{\text{earth}} = 6.38 \times 10^6 \text{ m}$
 $r_{\text{venus}} = 6.31 \times 10^6 \text{ m}$
 $r_{\text{mars}} = 3.42 \times 10^6 \text{ m}$

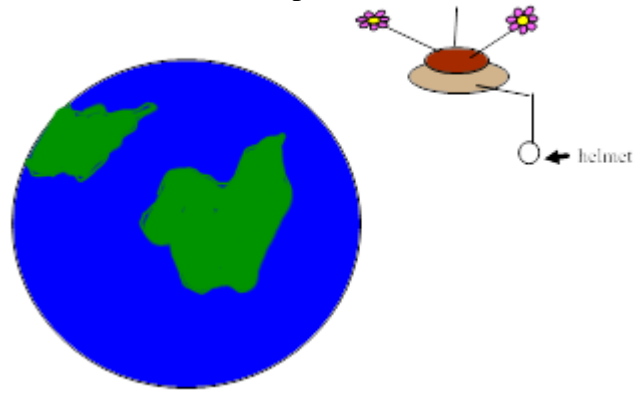
Example F.20 An object feels a gravitational force of 1000N on the Earth's surface. If the object were moved to a position above the earth that is exactly 1 full earth radius above the surface what amount of force would it feel?

Example E.21 Determine the earth's gravitational field intensity at: a) the Earth's surface b) at a height of 300km above the earth's surface.

Example F.22 What force would a 25kg mass feel on the surface of Venus?

Example F.23 Saturn has a gravitational field intensity of 10.4N/kg. a) Determine the radius. b) How high above the surface is the gravitational field intensity equal to that of Earth's at the surface?

Example F.24 You have been chosen to go on an amazing journey to do research on a new and yet to be named planet in a parallel solar system. Your assigned task is to determine the mass of this glorious new planet. You have the following tools: a fancy scale for measuring force, a **2kg test mass** (aka a space helmet), a telescope and a meter stick. Since there is no scale for measuring mass (and the planet wouldn't fit on most scales anyway) you need to find another way. Using your inherited intelligence and a sharp eye (along with the meter stick and the telescope) you determine the **radius of the planet is 2200km**. You also measure the force on the test mass and find a **force of 24.6N** pulling the helmet toward the surface of this unknown planet. All of these measurements are taken at a **height of 400km above the surface** of the planet. You may begin.



If you were to combine the two electrostatic field equations below you would get the equation for electrostatic force.

$$E = \frac{kq_s}{r^2} \qquad E = \frac{F_Q}{qr}$$

$$F_Q = \frac{kq_1q_2}{r^2}$$

The same can be applied to the two gravitational field intensity equations. The result will give you the gravitational force which is also called Newton's Law of Universal Gravitation.

$$g = \frac{F_g}{m} \qquad g = \frac{Gm_s}{r^2}$$

Newton's Law of Universal Gravitation

$$F_g = \frac{Gm_1 m_2}{r^2}$$

F_g = force of gravity (N)
 G = universal gravitational constant
 m_1 = first mass (kg)
 m_2 = second mass (kg)
 r = distance between centers of masses (m)

When working with a planet and a small object (relative to the planet) we use the following.

$$F_g = \frac{GMm}{r^2}$$

M = mass of planet (kg)
 m = mass of object (kg)

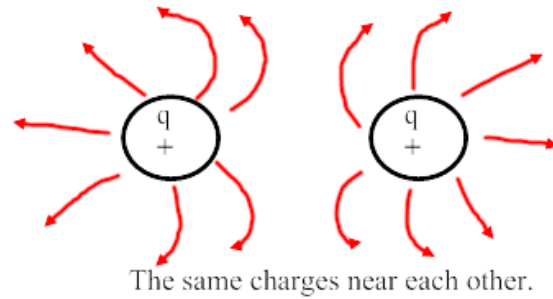
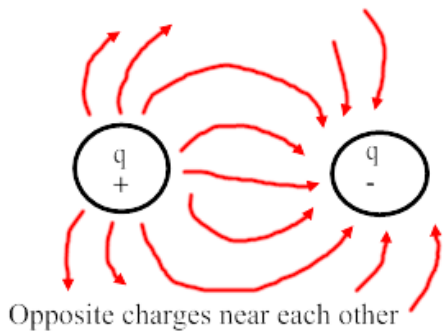
Example F.25 What force of gravity would a person feel if they were actually able to walk on the surface of Venus? Calculate the force if the person's mass is 100kg.

Field Lines

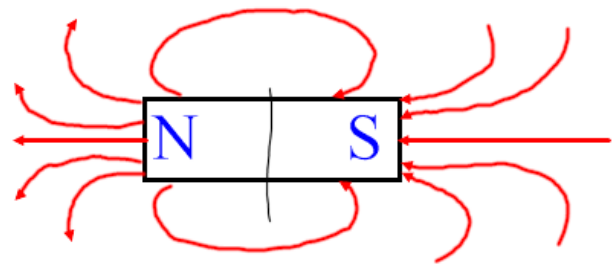
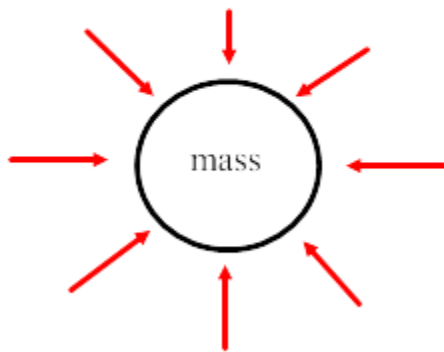
Electric

These are the field lines if each charge is by itself and not near anything





Gravitational

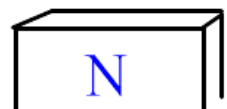
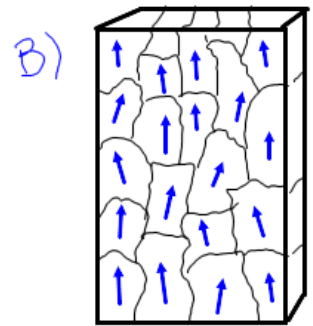
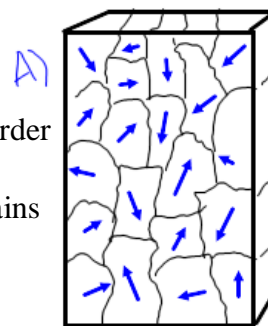


Magnetic

- a single magnet has North and South poles (North is like (+) and South is like (-))
- lines go from north to south
- all magnets possess a magnetic field
- the number of lines shows the strength of the magnet and is also called the magnetic flux
- the inverse square law still applies

Magnetic Domains

- all of the inner particles in a metallic material have magnetic properties called domains
- most metals have their domains arranged in a random order as shown in diagram A below
- if a permanent magnet is brought near a metal the domains can be rearranged to give the metal magnetic properties such as in diagram B (this is how you can magnetize a screwdriver to pick up loose screws, nuts, etc.



- if a permanent magnet is heated past a certain point it will lose its magnetism
- this temperature is called the Curie point and varies depending on the metal
- the following is just a few examples which you do not need to know

Material	Curie Point
iron	770 ⁰ C
cobalt	1131 ⁰ C
nickel	358 ⁰ C
gadolinium	16 ⁰ C

- if a magnet is heated it will weaken its strength but it will return to its original strength when cooled
- it only loses its magnetism if it is heated past the Curie point

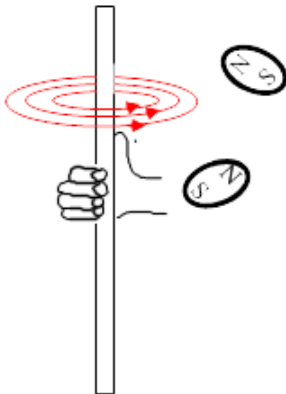
Oersted's Discovery and Electromagnetism

In 1819, Hans Christian Oersted discovered that a current carrying conductor will generate a magnetic field. His published findings led to the "new" phenomenon called electromagnetism. The flow of electrons will create a magnetic field. In reverse the movement of magnets over a coil of wire will generate a flow of electrons and create a current. This principle is what most forms of energy generation is based on. (Hydro, coal, nuclear and wind are all based on this principle) In any of these generators a turbine is forced to turn which in turn makes coils of wire pass over magnets creating the flow of electrons and electricity.

Based on this logic a rule was created called the Right Hand Rule (Left handed people can do it to but they have to use their right hand!) These rules come into play in most 1st year Physics courses in the second semester.

Right Hand Rule # 1

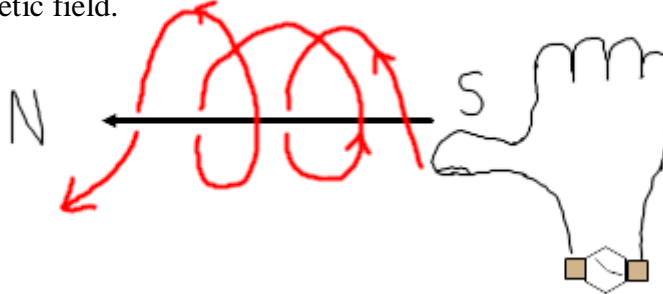
If you hold a current carrying conductor in your right hand with your thumb along the conductor, there will be a magnetic field around the conductor such as below.



Since wire is often wrapped in coils a second rule was created. Creativity was lacking at the time and it became known as the Right-Hand Rule # 2

Right Hand Rule #2

If you wrap your fingers in the direction of the current in the coil your thumb will point in the direction of the magnetic field.



Unit Summary

- Electrostatic fields and forces, gravitational fields and forces all follow the inverse square law
- don't put the negative in the equations for charged objects, use it to determine direction (repulsion or attraction)
- for multiple charged objects use vectors to solve for resultant force or field
- for electrostatic fields the field is determined with the assumption that the test charge at the point is positive
- for gravitational situations remember the distance (r) is always measured from the center of the object. This is also true for charged objects but the radius of the object is usually negligible
- make sure your distances are always in meters for the equations
- write down your givens, there are only 6 equations

Practice Problems

1. What is the gravitational field intensity at a height of 1.0Mm above the surface of Mars if an object, with a mass of 1.71slugs, feels a gravitational force of 93.75N on the surface? What is the direction of the field? **(2.19N/kg, inward)**
2. Three charged spheres are placed in a vertical line. The distance between each pair is 20cm. The first sphere has a charge of $12\mu\text{C}$, the second has a charge of $20\mu\text{C}$ and the third has an unknown charge. The resultant force is +6N. What is the charge of the unknown sphere? **(13.33 μC)**
3. A small sphere is given a charge of $-37\mu\text{C}$, and a second identical sphere is given a charge of $+17\mu\text{C}$. If the two spheres are 10 inches apart, find the force between the two spheres. **(87.75N)**
4. If the two spheres in question 3 are allowed to touch and then separated, what force will exist between the two spheres when they are 20 inches apart? **(3.49N)**
5. What is the field intensity and direction directly in the middle of two charged sources that are 50cm apart? The left side source has a charge of $-55\mu\text{C}$ and the right side source has a charge of $-60\mu\text{C}$. **(0.72MN/C at 0°)**
6. Three charged spheres are located at the vertices of a right triangle. Charge A ($76\mu\text{C}$) is at (0,80); charge B ($96\mu\text{C}$) is at (0,0); charge C ($-80\mu\text{C}$) is at (-60,0). All distances are in centimeters. Determine the resultant force on A. **(67N at 119.16°)**

7. Determine the magnitude and direction of the electric field intensity at point P given the following coordinates :

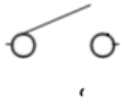
Point P (20,20)

Source 1: $-50\mu\text{C}$ (-10,0)

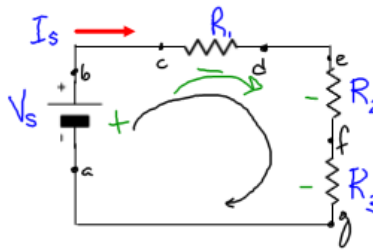
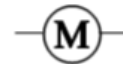
Source 2: $+25\mu\text{C}$ (40,5)

(Show all you work) (**147N at 322.74°**)

8. An object has a weight of 160N when it is at a location exactly one full Mars radius away from the surface of Mars. What is its weight at a location exactly one full Saturn radius from the surface of Saturn? (**457N**)
9. An object near the surface of Saturn would experience an acceleration of gravity of 10.4m/s^2 . a) What is the mass of an object that weighs 800N on the surface of Saturn? b) What would be the mass of the same object on Earth? c) What would the mass be in slugs? (**76.92kg, 76.92kg, 5.25 slugs**)
10. You have become the first person in the world to reach of wonderful and glorious new planet (which will be named after me!). You have decided to put some of this Physics stuff to use. You drop a 3.0kg ball onto the surface and it takes 2.60 seconds to cover a distance of 6 feet. The planet has already been determined to have a radius of 8.40 million meters (making it bigger than Pluto and therefore an official planet). What is the mass of the planet? (**$5.71 \times 10^{23}\text{kg}$**)
11. Henry has decided to throw a baseball hard enough to put into orbit by throwing it horizontally, tangent to Earth's surface. He is standing on top of a small hill that is 97km above the Earth's surface. With what speed must he throw the ball to put into orbit? (**7.86km/s**)
12. The ISS (International Space Station) orbits the Earth at an altitude of approximately 410km. It has a mass of 450 000kg. a) Determine the average speed of the ISS required to keep it in orbit. b) Determine the period of its orbit. (**7664m/s, 1.55h**)
13. A negative charge of $10\mu\text{C}$ and mass $5\mu\text{kg}$ orbits a massive positive charge of $26\mu\text{C}$ in a circular orbit of radius 5 meters. To the nearest tenth of a m/s what is its speed? (**0.68m/s**)
14. Two pith balls that are equally charged and each has a mass of 2.5 g. Both balls are suspended by thread. Both pith balls hang in a way that equilibrium is reached. At this point the balls are 3.2cm apart and the angle between the balls is 50° . A) Calculate the force on each pith ball. B) Calculate the charge on each pith ball. (Determine the forces on the pith ball and create a triangle using the force vectors) c) Determine the force of tension in each thread.
15. The moon is about 382500 km from the Earth (give or take depending on its orbit). Would a 1000kg object be pulled to the moon or the Earth if it is $3.725 \times 10^8\text{m}$ from the Earth's surface? (**moon**)
16. A sphere with a charge of $40\mu\text{C}$ is used in question ? of assignment # ? to create equilibrium. What are the coordinates of its location? (**47, 32**) (**question number and assignment # TBD**)



Electricity



Unit 2 - Electric Energy and Electric Circuits

Definitions (on website)

conductor - a material that allows electric charges to flow easily

insulator - a material that doesn't allow electric charges to flow easily

electrostatics - the study of electric charges at rest (unit 1)

voltaic cell - a cell containing two different metals, called electrodes, placed in an electrolytic solution that produces an electric charge on the electrodes
- provided scientists with the first chance to work with a flow of electrons rather than static electricity

Alessandro Volta - created the first voltaic cell

battery - a combination of two or more voltaic cells

electrodes - an electrical conductor through which a current enters or leaves and electrical device

anode - the positive pole of a primary cell or battery

cathode - the negative electrode of a cell or battery

Electric Potential Difference

Electric potential difference between two points is defined as the change in electrical energy divided by the amount of charge that passes between those two points. The electrical potential difference is also referred to as the voltage or voltage drop across two points.

$$V = \frac{\Delta E_Q}{q}$$

V = electric potential difference (aka voltage) (volts)

ΔE_Q = change in potential difference (J)

q = quantity of charge (C)

* E_Q is equal to the amount of work done on an object

Example 2.1 A battery has an electric potential difference of 12V. How much work is done when 48C of charge moves from the anode to the cathode?

Given
V = 12V
 $E_Q = ?$
q = 48C

$$V = \frac{E_Q}{q}$$
$$12 = \frac{E_Q}{48}$$
$$E_Q = 576J$$

The work done is equal to the amount of energy between the two points which is 576J.

Electric Current

Electric current is defined as the amount of charge passing a point over a given amount of time.

$$I = \frac{q}{t}$$

I = current (A = amps, amperes)
q = amount of charge (C)
t = time (s)

Example 2.2 Your house has an electrical system that operates at 120V. A toaster draws 9.6A for 2.5 minutes. a) Find the amount of charge passing through the toaster. b) Find the amount of electrical energy that the toaster converted to heat energy.

Given

$$V = 120V$$

$$I = 9.6A$$

$$t = 2.5\text{min} = 150 \text{ sec}$$

$$q = ?$$

$$E_Q = ?$$

$$\begin{aligned} \text{a) } I &= \frac{q}{t} \\ 9.6 &= \frac{q}{150} \\ q &= 1440C \end{aligned}$$

$$\begin{aligned} \text{b) } V &= \frac{E_Q}{q} \\ 120 &= \frac{E_Q}{1440} \\ E_Q &= 172800J \end{aligned}$$

While the toaster was on 1440C of charge passed through it and it converted 1.73×10^5 J of electrical energy to heat energy.

Example 2.3 An electric heater is turned on in your house for 30 minutes. If 3.0MJ of heat energy is generated, how much current is drawn by the heater?

Conventional Flow vs. Electron Flow

Electron flow is the flow of negative charge (electrons) from the cathode (-) to the anode (+). However, since scientists couldn't see electrons when they first started making electrical discoveries they believed that current went from (+) to (-) which is called conventional current. Since so much theory was developed and so much text was written using this method it was kept and is still used today. Even though the actual direction is different it makes no difference to the calculations for electricity.

Resistance to Flow of Charge

- basically it is the amount of friction encountered by a charge as it flows through a circuit
- the longer the length of circuit, the greater the resistance
- the smaller the cross-sectional area (diameter of a round conductor) the greater the resistance

$$R = \frac{\rho L}{A}$$

R = resistance (Ω - ohms)
 ρ = resistivity (Ω m)
(see table below)
L = length of conductor (m)
A = cross-sectional area (m^2)
* ρ = rho
* Ω = omega

Remember the equation for area from math

$$A = \pi r^2 \quad (\text{for circular conductors})$$

or

$$A = \frac{\pi d^2}{4} \quad (\text{if using the diameter})$$

$$A = l \times w \quad (\text{for rectangular conductors})$$

Resistivity of a conductor is based on temperature. The greater the temperature the greater the resistance.

Resistivity Table

Material	Resistivity(Ω m)
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminum	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Nichrome	100×10^{-8}
Carbon	3500×10^{-8}
Germanium	0.46
Glass	1×10^{10} to 1×10^{14}

In the electrical trade wires are usually rated for their thickness by the term gauge. The higher the number the thinner the wire. Houses are generally wired with 14 gauge. The gauge concept also applies to sheet metal.

Some common Gauges and Their Diameters (Note: we use radius in our calculations)

Gauge	Diameter (mm)
0	9.35
10	2.59
14	1.63
18	1.02
22	0.64

Example 2.4 How much resistance would you expect to find in an 10 gauge extension cord that is 15m long if

- the wire is made of silver?
- the wire is made of copper?

Ohm's Law

In 1826, George Simon Ohm determined, after lengthy experimentation, that the voltage across a load (anything with electrical resistance) is equal to the current multiplied by the resistance of the load.

$$V = IR$$

V = voltage (V)

I = current (A) (A = amps)

R = resistance (Ω)

Note: It should be noted that this law only applies to metal conductors at constant temperature. Once it is heated it doesn't follow Ohm's Law.

Example 2.5 Determine the resistance of a load in a circuit if the current across the load is 5A and the voltage supplied to the load is 110V.

Given

$$I = 5A$$

$$V = 110V$$

$$R = ?$$

$$V = IR$$

$$110 = 5R$$

$$R = 22\Omega$$

The resistance is 22Ω .

Electric Power and Energy

Every appliance has/should have a power rating that tells the amount of power it uses at its maximum setting. For example

- a stove might be - 12000W
- a lamp might be - 150W
- a clothes dryer - 5000W
- clock - 5W
- light bulb - 60W, 100W or if they are CFL (compact fluorescent lights -10W, 13W)

To determine the power there are 3 different equations which are all variations of each other using rearrangements of Ohm's Law.

$$P = VI$$

P = power (W-watts)

V = voltage (V)

I = current (A)

Example 2.6 A fan is rated to be used with a 120V electrical system that is standard in North America and it draws 0.45A at its maximum setting. a) What is the power of the fan motor? b) What is the resistance of the fan motor?

Given

$$V = 120V$$

$$I = 0.45A$$

$$P = ?$$

$$R = ?$$

$$\begin{aligned} \text{a) } P &= VI \\ P &= 120(0.45) \\ P &= 54W \end{aligned}$$

$$\begin{aligned} \text{b) } V &= IR \\ 120 &= 0.45R \\ R &= 266.67\Omega \end{aligned}$$

The fan has a resistance of 266.67Ω while having a power of 54W.

Household Voltage - North America vs. Europe

In North America most household appliances are designed to operate at 120V of voltage. In Europe, household appliances are designed to operate at 240V. An appliance designed to work in North America will draw a certain current. If that appliance is taken to Europe the voltage it receives is increased and as a result so is the current. The excess current causes the circuit in the appliance to heat up and possibly breaking the circuit rendering the appliance useless. You should note that the only thing that stays the same in the appliances is the resistance since this is a feature that is a fixed property of the appliance.

Example 2.7 A lamp was designed for use in North America and has a rating of 150W when used with a 120V source. What would be the power output in Europe?

Version 2 of Power Equation

Combine the following equations and replace I

$$\begin{array}{ll} P = VI & V = IR \\ P = V\left(\frac{V}{R}\right) & \frac{V}{R} = I \\ P = \frac{V^2}{R} & \end{array}$$

$$P = \frac{V^2}{R}$$

P = Power (W)
V = Volts (V)
R = Resistance (Ω)

Example 2.8 A heater is built in Europe and has a power rating of 1800W. What power would it generate if used in Canada?

Combine the following equations but replace V

$$\begin{array}{ll} P = VI & V = IR \\ P = (IR)I & \\ P = I^2R & \end{array}$$

$$P = I^2R$$

P = power (W)
I = current (A)
R = resistance (Ω)

Example 2.09 Above it was stated that the current doubles when switching between the two continents. Using the power outputs of 1800W in Europe and 450W in Canada from the previous example prove that the current doubles using the newest equation.

Electrical Energy

If you remember from Physics 11 we talked about Power. We said it was equal to the amount of work done divided by the time. In equation form it was:

$$P = \frac{W}{t}$$

You will also remember that work and energy can be interchangeable. Therefore we can rearrange to get to the following equation for energy or in our case electrical energy:

$E_q = Pt$	$E_q = \text{electrical energy (J)}$ $P = \text{power (W)}$ $t = \text{time (sec)}$ (* for calculating electrical cost we use hours)
------------	---

Example 2.10 Determine the power and current of a kettle that converts 72KJ of electrical energy to heat energy over a period of 2 minutes.

Cost of Electricity

Customers of the Electrical company pay a rate based on the number of kWh of electrical energy used. Note the energy is not in joules because the energy is measured in hours of use not in seconds. The current cost of energy in New Brunswick is 9.54¢/kWh for the first 1300kWh used and then 8.61¢/kWh. New dwellings will be charged a flat rate of 9.20¢/kWh.

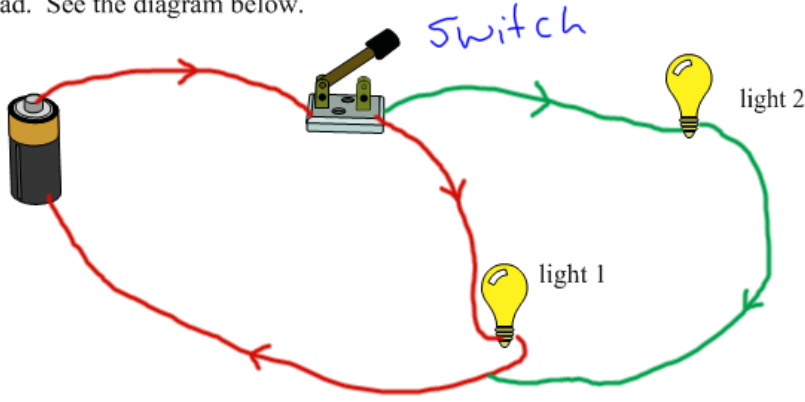
$\text{Cost} = E_q \times \text{rate}$	$\text{Cost} = \text{cost of electricity (Cents)}$ $E_q = \text{energy (kWh)}$ $\text{rate} = \text{cost} / \text{kWh}$
--	---

Example 2.11 How much would it cost to operate two desk fans for four hours. The fans operate on 120V and draw 0.4A.

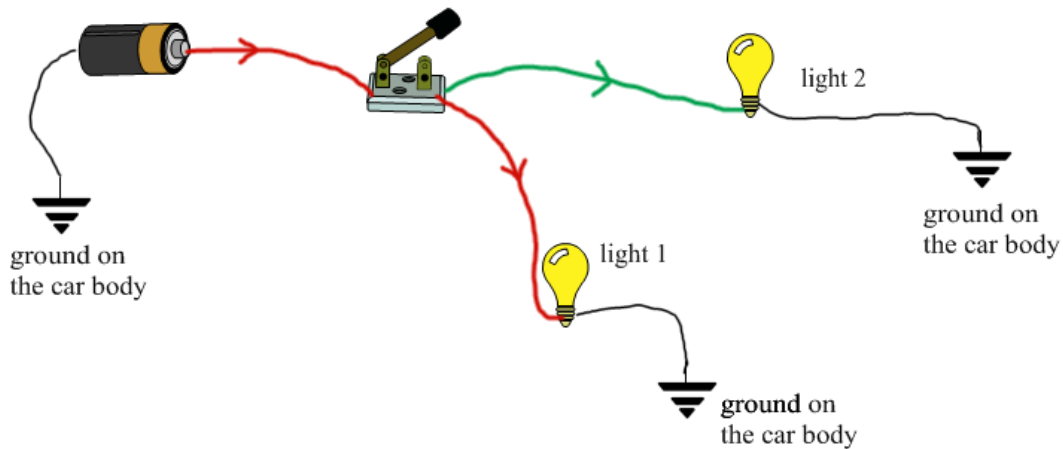
Example 2.12 A floor heater is turned on in a room and left to heat the room. If it is turned on to 80% and has a maximum power rating of 1800W. a) How long can it be left on if the cost of operating it was \$1? b) What is the current draw?

Ground Systems in Automobiles

For a basic direct current electrical circuit we would connect a wire to the positive post of a battery. The wire would go to the load and then another wire would go from the other side of the load and back to the battery. The process would be repeated for each additional load. See the diagram below.

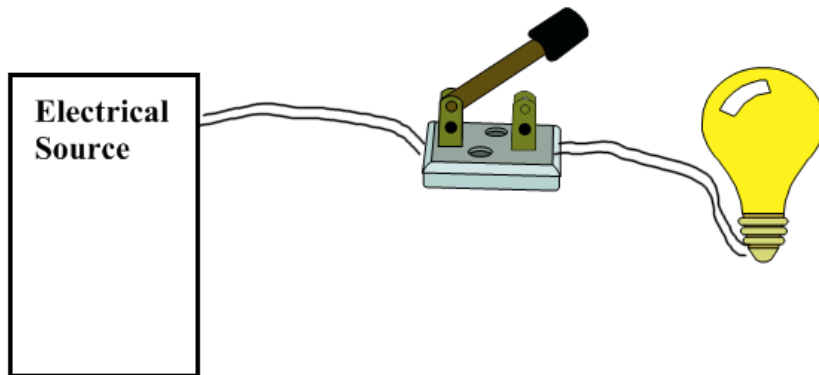


In a car, especially with a lot of accessories, there needed to be a more efficient method. As such a ground system was developed where the body of the vehicle acts as one large ground wire to allow the electrical charge to return to the battery. See the diagram below.



Residential Wiring

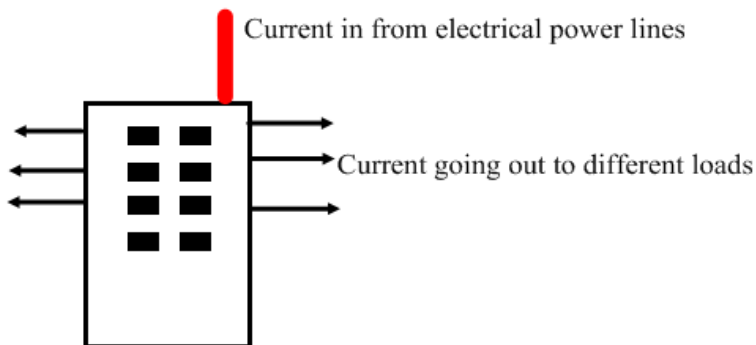
Wiring in homes is different than in cars in a couple ways. The first way is that in cars the electricity flows in one direction and is called DC (direct current). In a house the electricity operates on AC (alternating current). For AC the current is alternating there is a need to run wire back to the source but it appears as if only one wire goes back because two wires are incased inside the insulating plastic.



Circuit Breakers/Fuses

Every house and car has one or the other of these. It serves several purposes.

1. It acts as power distribution center
2. It helps limit the power that goes to each circuit
3. Acts as a safety device



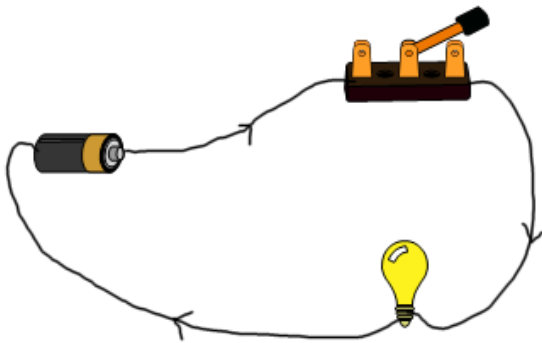
Houses can have a circuit breaker which is a switch that will flip off to 'break' the circuit if too much current passes through the circuit. Houses may also have fuse panel which is what a car has too. Fuse panels work on the same principle except that a fuse will 'blow' if too much current goes through it.

What would happen if too much current goes through a circuit.

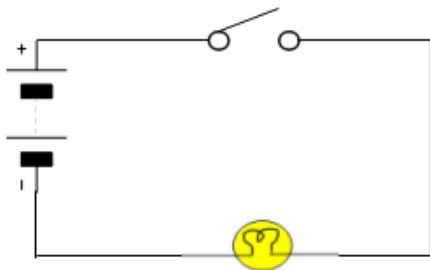
Electrical Circuits

- a circuit is an electrical path comprised of electrical components such as switches, power sources and loads
- loads are anything that draws a current (resistors, light bulbs, motors, etc)
- power sources are such things as cells, batteries or AC or DC generators
- switches are used to open or close a circuit
- an open circuit has a break or opening in the circuit (current won't flow)
- a closed circuit has all connections complete (current will flow)

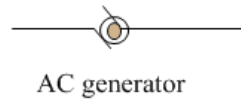
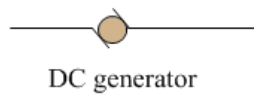
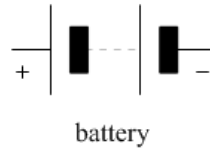
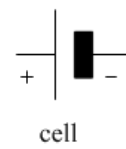
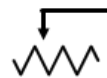
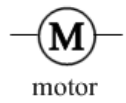
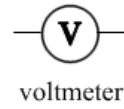
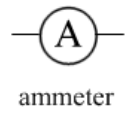
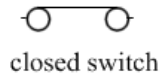
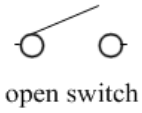
Drawing Circuits



Because this is time consuming and cumbersome so a more standardized method was created to simplify the drawings. The above diagram would look as follows.

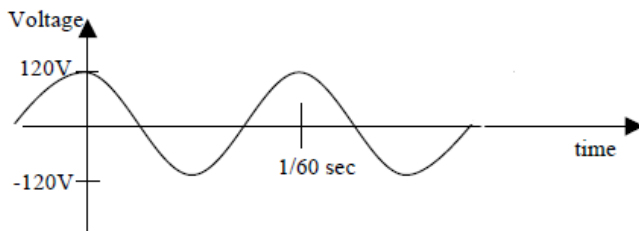


Symbols used for Circuit Analysis (Yes, you need to know them)



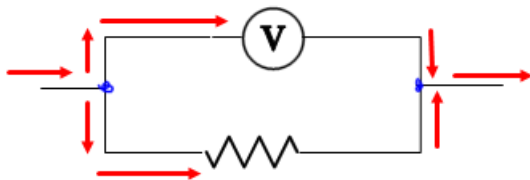
DC Power: Voltage and current flow in one direction. Examples are batteries and the power supplies we use in class.

• **AC Power:** Voltage and current flow in alternate directions. In the US they reverse direction 60 times a second. (This is a more efficient way to transport electricity and electrical devices do not care which way it flows as long as current is flowing. Note: your TV and computer screen are actually flickering 60 times a second due to the alternating current that comes out of household plugs. Our eyesight does not work this fast, so we never notice it. However, if you film a TV or computer screen the effect is observable due to the mismatched frame rates of the camera and TV screen.) Electrical current coming out of your plug is an example.



Ammeters and Voltmeters

- volt meters measure voltage and are connected in parallel
- parallel connection means 2 or more elements are beside/parallel to each other



- ammeters measure current (amps) and are connected in series
- series connection means each element is connected one after the other in a line or series



Circuit Analysis

Three Types

1. Series
2. Parallel
3. Complex (combination of series and parallel)

For all circuits we are trying to simplify to get the circuit reduced down to one equivalent resistor. We will be using a variation of $V = IR$

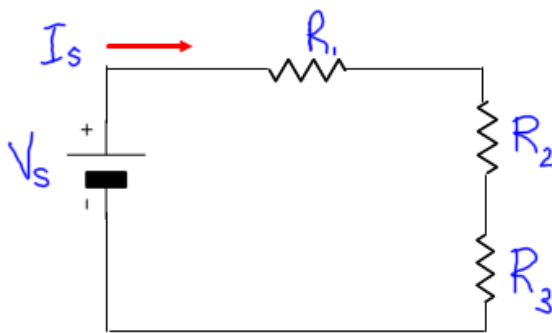
$$V_s = I_s R_{eq}$$

V_s = voltage from the source (V)

I_s = current from the source (A)

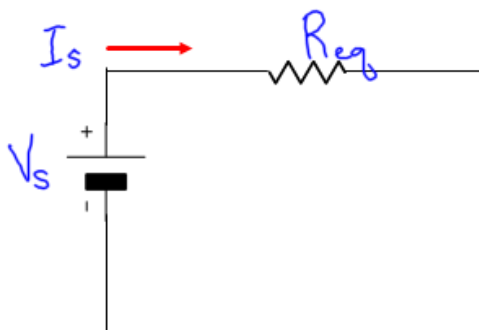
R_{eq} = equivalent resistance of the circuit (Ω)

Original Circuit



*We will simplify the resistors in the original circuit down to one resistor as shown in the simplified circuit based on the rules for the different types of circuits

Simplified Circuit



* In a series circuit there is only one circuit

There are couple ways to solve circuits.

1. You can use the equations for equivalent resistance and work from there
2. You can solve for each load using $V=IR$ for each one

Either way we can draw simplified circuits to get to the point where there is only one resistor to help solve most problems. See diagram below. You will need to draw simplified circuits for all complex circuits

Series Circuit

In a series circuit the current has only one path and must pass through all loads

Series Circuit Rules

1. Voltage from the source drops across each resistor. The voltage across each resistor adds up to equal the voltage from the source

$$V_s = V_1 + V_2 + V_3 + \dots V_n$$

2. All the current in the circuit passes through each resistor (\therefore current stays the same)

$$I_s = I_1 = I_2 = I_3 = \dots I_n$$

3. The equivalent resistance in a circuit is equal to the sum of all the resistors in the circuit

$$R_{eq} = R_1 + R_2 + R_3 + \dots R_n$$

Example 2.13 A series circuit has 4 resistors (2Ω , 6Ω , 4Ω and 3Ω) and a 12V source.

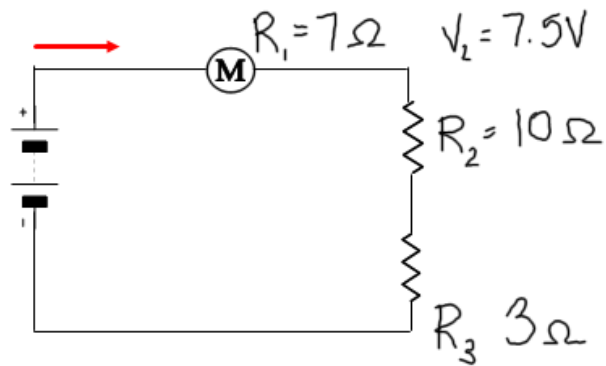
- a) Draw the circuit
- b) Determine the R_{eq} for the circuit
- c) Determine total current in the system
- d) Determine the voltage across each load

Example 2.14 A circuit is composed of a battery with 3 loads as shown below.

The voltage across the 10Ω resistor is $7.5V$

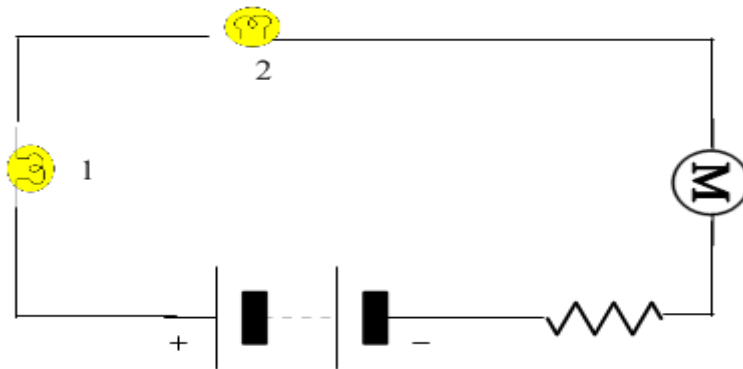
a) Find the equivalent resistance

b) Find the voltage from the source

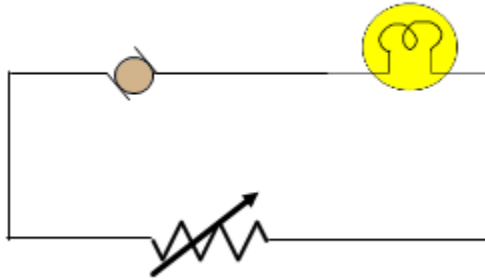


·Example 2,15 A series circuit is comprised of 2 different lights, a resistor and a small motor. The first light has a resistance of 12Ω , the second light has a resistance of 10Ω and the motor has an unknown resistance. The resistance of the resistor is also unknown. The power for the second light is $10W$. Based on this information determine:

- The source current
- The resistance of the motor if the power is $20W$
- The resistance of the resistor if the source voltage is $60V$.
- The work done by the motor if it is left on for 10 minutes



Example 2.16 A series circuit is designed with a light bulb and a potentiometer. The source voltage is 120V and the resistance is 15Ω . What would you have to set the 'pot' at to get the power of the light to be 60W?



Parallel Circuit

In a parallel circuit the current has a choice of paths and doesn't pass through all loads

Parallel Circuit Rules

1. Voltage from the source is the same across each possible path and resistor.

$$V_s = V_1 = V_2 = V_3 = \dots V_n$$

2. The current entering a junction equals the current leaving the junction

$$I_s = I_1 + I_2 + I_3 + \dots I_n$$

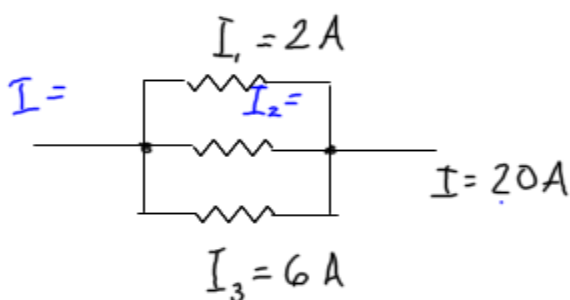
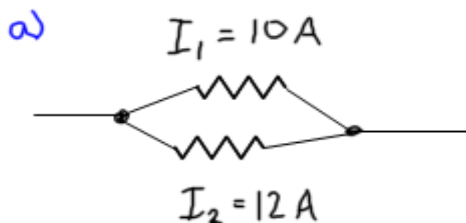
(see the following example)

3. The equivalent resistance in a circuit is determined using the following equation

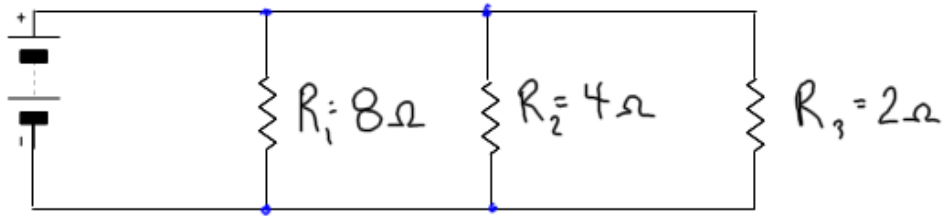
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_n}$$

To be able to determine different currents we need to understand node concept, also called nodal analysis. It says that **all the current entering a node or junction is equal to all the current leaving.** (This is also part of Kirchoff's law)

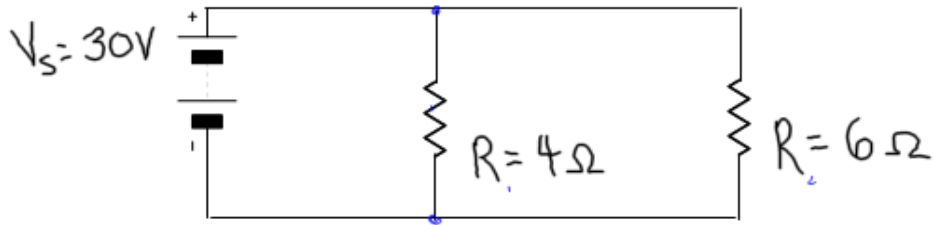
Example 2.17 Determine the unknown currents in the following diagrams



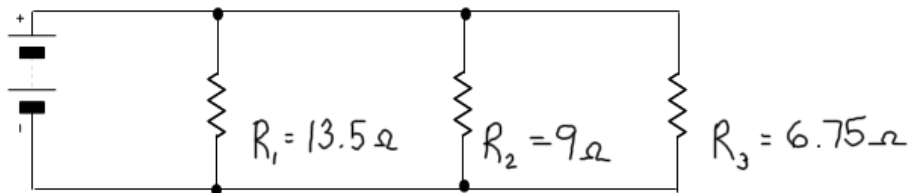
Example 2.18 Determine the equivalent resistance for the following circuit.



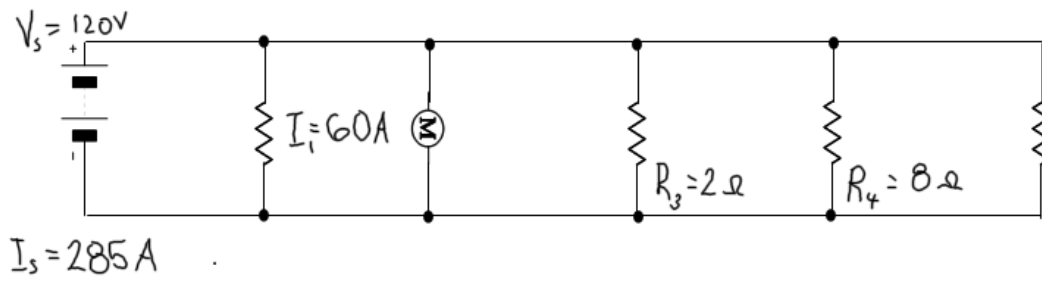
Example 2.19 Solve the following parallel circuit for the missing values.



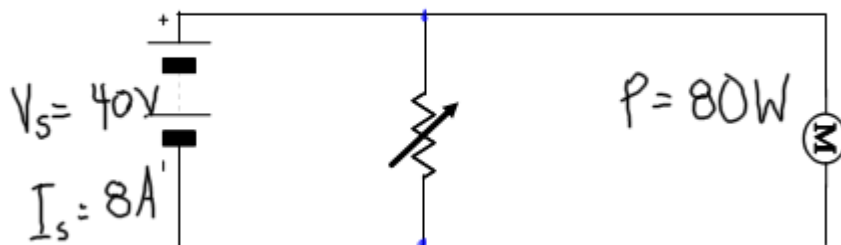
Example 2.20 Determine the missing variables for the following circuit. The current through R_2 is 2A.



Example 2.21 Solve for the unknowns in the following circuit.



Example 2.22 In the following circuit determine the resistance you would set the pot at to make the motor have a power rating of 80W. If you increased the resistance in the pot would the power of the motor increase or decrease?



Complex Circuits

- combination of series and parallel circuits

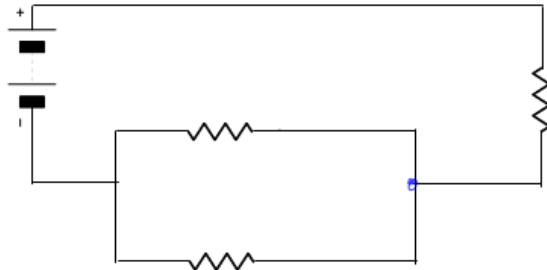
Remember

- Series - voltage changes
- current constant
- Parallel - voltage constant
- current changes

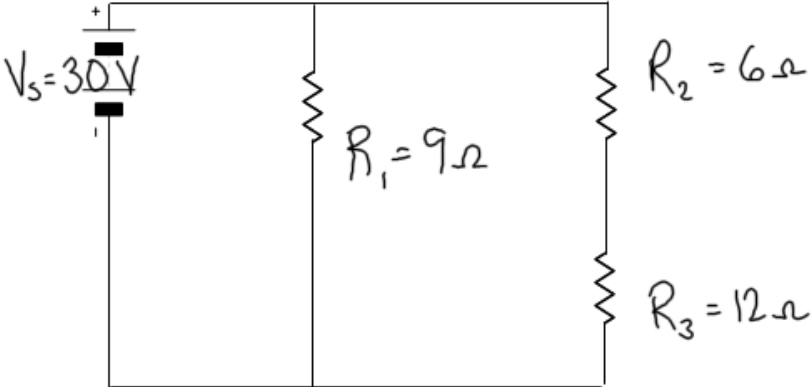
Simplifying - no specific method (**you have to draw simplified circuits** to simplify it down either a series or parallel circuit)

- you must look at the circuit and determine whether it looks like a series or a parallel circuit
- if it seems like a series - simplify all the parallel parts first
- if it seems like a parallel circuit - simplify the series parts first

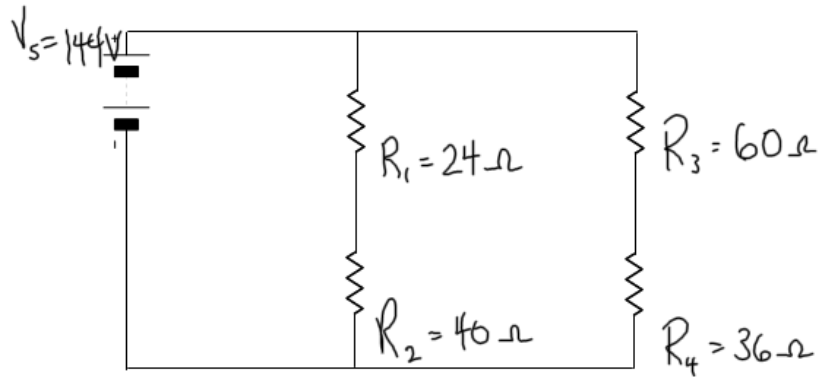
Example 2.23 Find the missing values. Simplify the circuit until it is a series circuit or a parallel circuit.



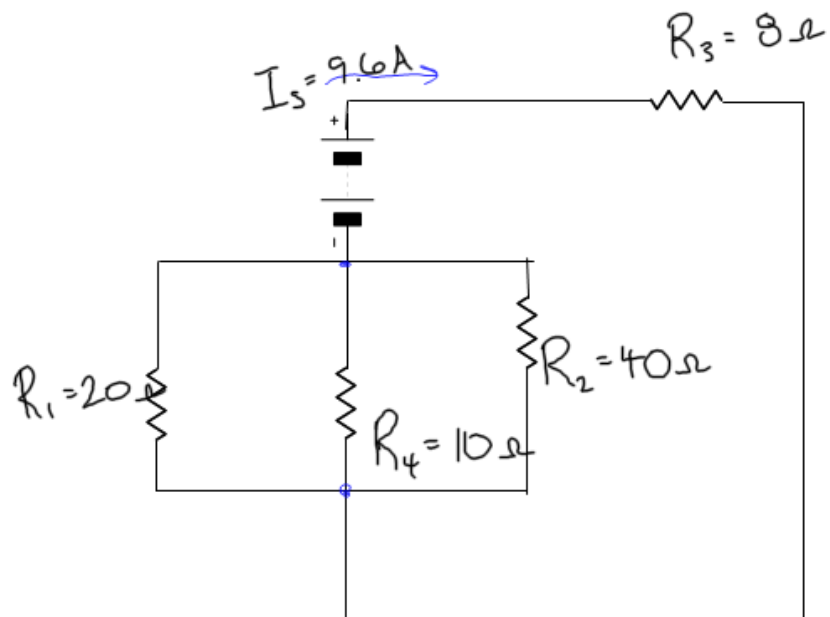
Example 2.24 Determine the missing values in the following circuit that Julio made.



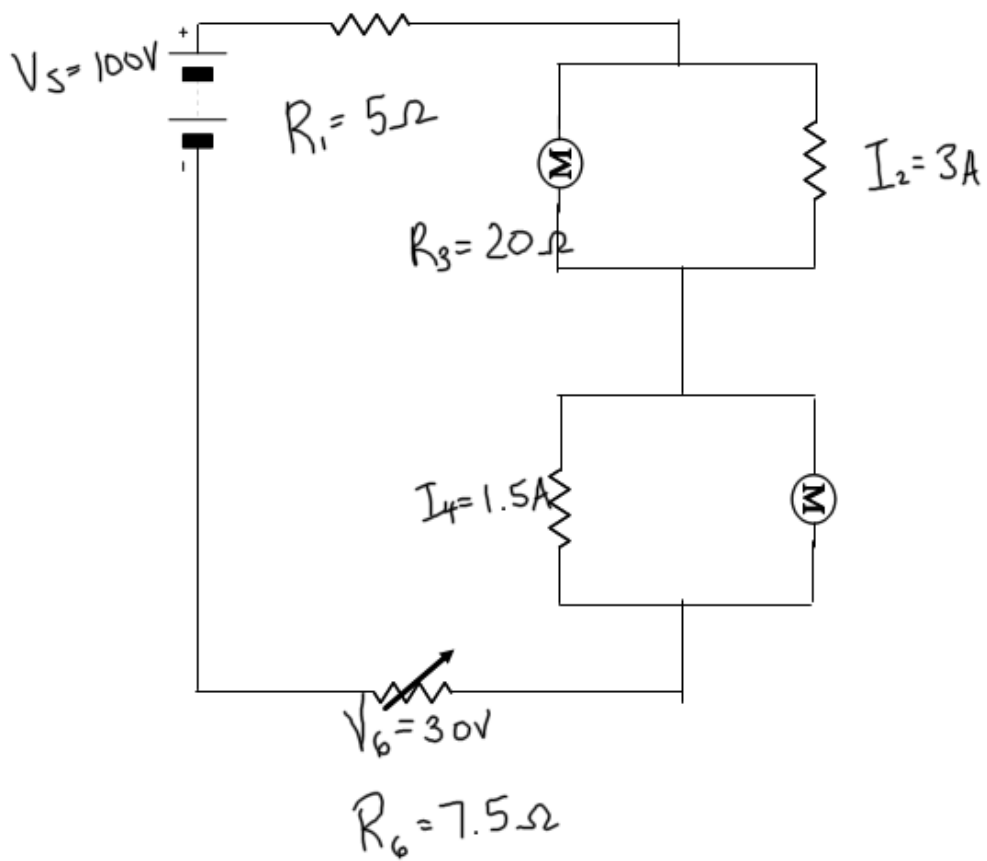
Example 2.25 Determine the unknown values in the following circuit, lovingly designed by José Guido Santore. April Fool's (or therest aboutest), t'was actually I, Mr. Gaunce, that created it. Thoughtst, you shall be able to completeth this circuit.



Example 2.26 Solve the following nifty circuit for the unknown values.



Example 2.27 Determine the power of each motor in the circuit below.



Kirchoff's Rules

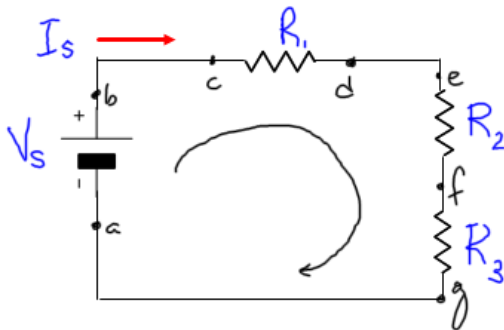
1st Rule - Junction Rule - the sum of all the currents entering a junction or node must equal the sum of the currents leaving the node or junction

2nd Rule - Loop rule - The algebraic sum of changes in potential around any closed circuit path(loop) must be zero.

Note - for the second rule sources are positive and voltages across resistors (drops) are negative when the current goes in a normal conventional direction. It is the opposite if you go in the reverse order.

These two rules can be used for any circuit but we only need to use it for certain complex circuits.

Example



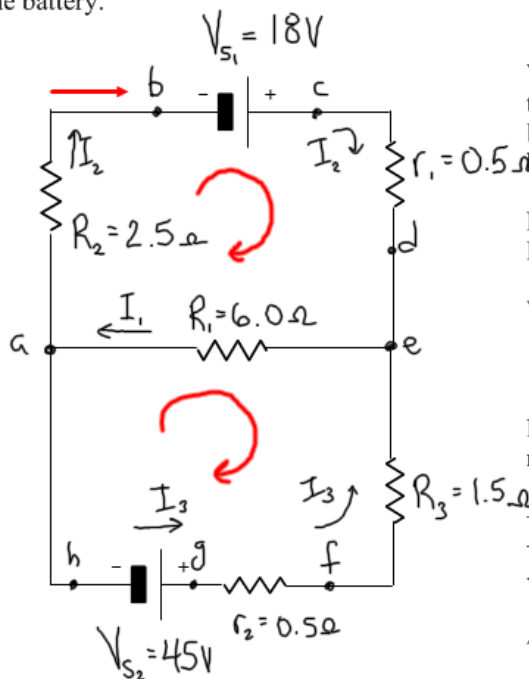
If we look at the loop **abcdefg** and apply the 2nd rule in the clockwise direction it would be:

$$V_s - V_1 - V_2 - V_3 = 0$$

$$V_s - I_1R_1 - I_2R_2 - I_3R_3 = 0$$

If we had the voltage and the current we could solve for missing values.

Let's look at one that is more difficult. This example is one that can't be simplified by the series-parallel method we have been using. The lower case r represents internal resistance of the battery.



We put the currents on in the direction we think that it goes. We will look at the top loop first and apply the 2nd rule. I_1 is different because it is shared by both loops.

Loop abcdea (Replace V with the appropriate IR equation)

$$V_s - I_2r_1 - I_1R_1 - I_2R_2 = 0 \quad (\text{Sub in values})$$

$$18 - I_2(0.5) - I_1(6.0) - I_2(2.5) = 0$$

$$18 - 3I_2 - 6I_1 = 0 \quad (\text{eq. 1})$$

Loop aefgha (We will stick to clockwise to maintain consistency)

$$+I_1R_1 + I_3R_3 + I_3r_2 - V_{s2} = 0$$

$$+I_1(6.0) + I_3(1.5) + I_3(0.5) - 45 = 0$$

$$-45 + 6I_1 + 2I_3 = 0 \quad (\text{eq. 2})$$

Apply the 1st Law at a point e

$$I_1 = I_2 + I_3 \quad (\text{eq. 3})$$

We now have 3 equations and 3 unknowns. We can rearrange and use systems of equations to solve for each current.

Replace I_1 in (1) by substituting (3) in (1)

$$\begin{aligned} 18 - 3I_2 - 6(I_2 + I_3) &= 0 \\ 18 - 3I_2 - 6I_2 + 6I_3 &= 0 \\ 18 - 9I_2 - 6I_3 &= 0 \quad (4) \end{aligned}$$

Replace I_1 in (2) by substituting (3) in (1)

$$\begin{aligned} -45 + 6(I_2 + I_3) + 2I_3 &= 0 \\ -45 + 6I_2 + 6I_3 + 2I_3 &= 0 \\ -45 + 6I_2 + 8I_3 &= 0 \quad (5) \end{aligned}$$

Solve for I_2 or I_3 by combining (4) and (5)

We will solve for I_2 by multiplying (4) by 2 and (5) by 3

$$\begin{aligned} 18 - 9I_2 - 6I_3 &= 0 \quad \times 2 & -45 + 6I_2 + 8I_3 &= 0 \quad \times 3 \\ 36 - 18I_2 - 12I_3 &= 0 \quad (6) & -135 + 18I_2 + 24I_3 &= 0 \quad (7) \end{aligned}$$

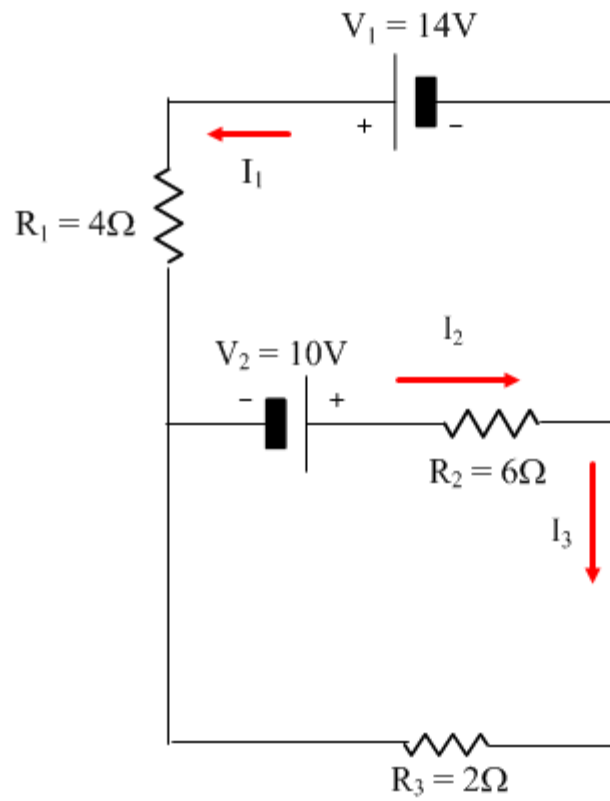
$$\begin{aligned} 36 - 18I_2 - 12I_3 &= 0 \quad (6) \\ -135 + 18I_2 + 24I_3 &= 0 \quad (7) \\ \hline -99 + 12I_3 &= 0 \\ I_3 &= 8.25\text{A} \end{aligned}$$

$$\begin{aligned} \text{Sub } I_3 = 8.25 \text{ into (2)} \\ -45 + 6I_1 + 2I_3 &= 0 \\ -45 + 6I_1 + 2(8.25) &= 0 \\ 6I_1 - 28.5 &= 0 \\ I_1 &= 4.75\text{A} \end{aligned}$$

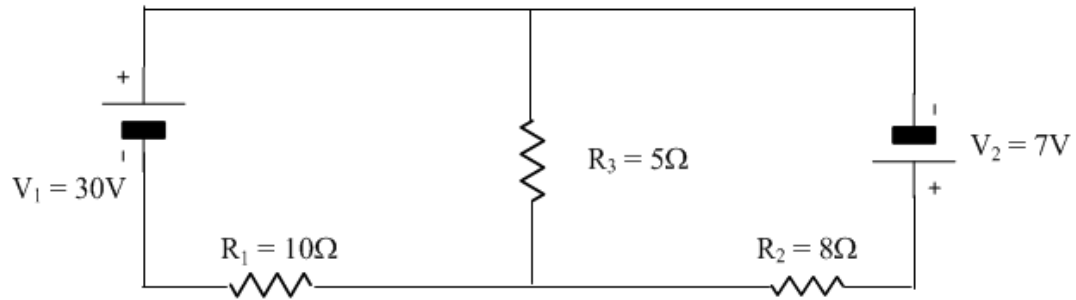
$$\begin{aligned} \text{Sub } I_1 = 4.75 \text{ and } I_3 = 8.25 \\ \text{into (3)} \\ I_1 &= I_2 + I_3 \\ 4.75 &= 8.25 + I_3 \\ I_3 &= -3.5\text{A} \end{aligned}$$

Since I_3 is negative it means we assumed the wrong direction of I_3 . Checking the circuit will show that the values for the currents are correct.

Example 2.29: Determine the current in the following circuit.

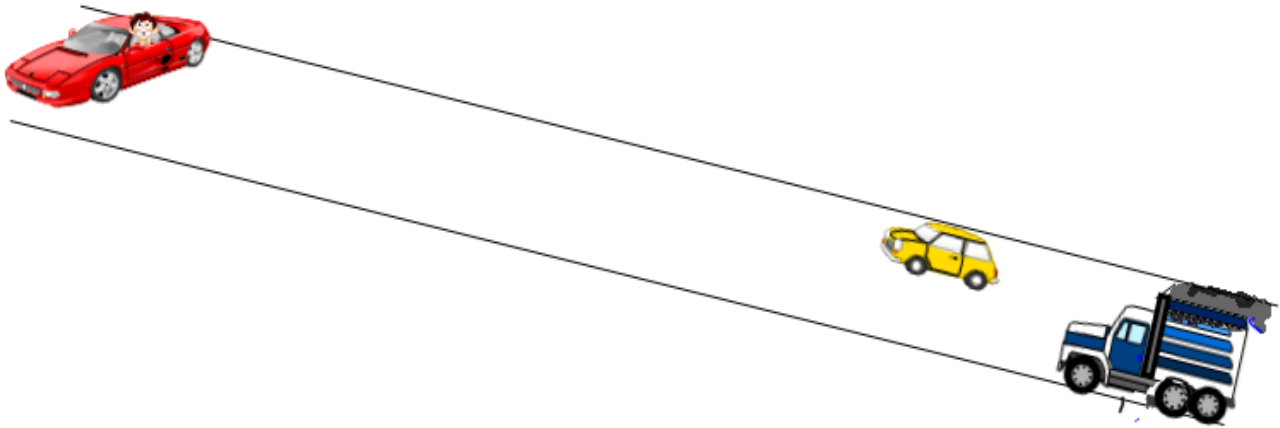


Example 2.30 Determine the currents for the following circuit.





Motion



Motion - Review and Extension

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$v = v_0 + a(t - t_0)$$

$$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

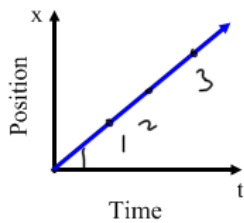
$$x = x_0 + vt - \frac{1}{2}a(t - t_0)^2$$

$$x = x_0 + vt$$

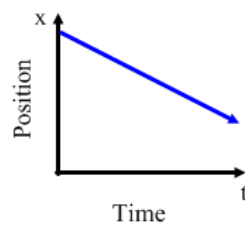
Graphs - Position, Velocity and Acceleration with respect to Time

1. Constant Velocity - straight lines on a Position-time graph

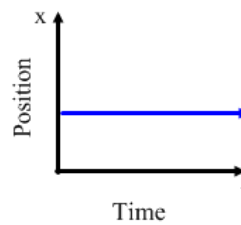
Moving east



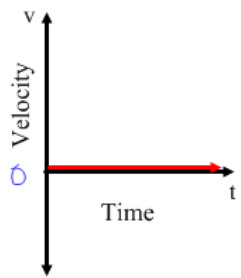
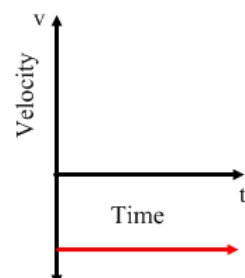
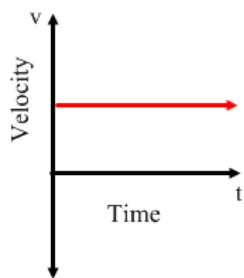
Moving west



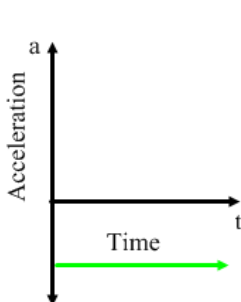
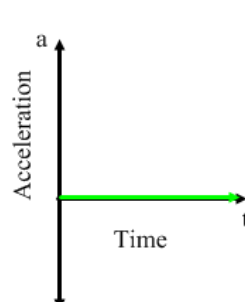
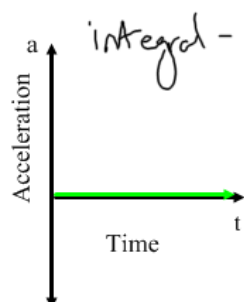
Not moving



We can create a graph of velocity vs. time by taking the slope of the position vs. time graph. (The derivative of the position equation if you have the equation)



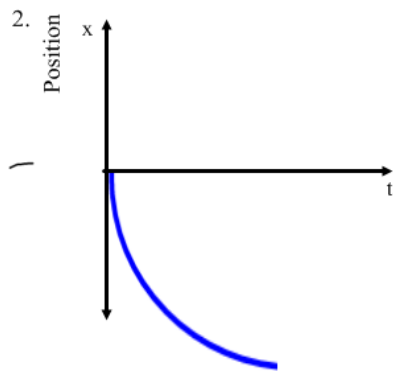
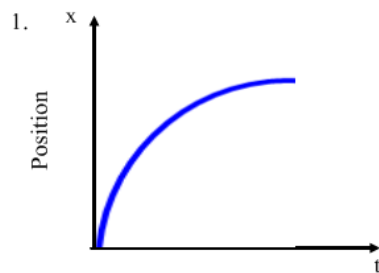
You can create a graph of acceleration vs. time by taking the slope from a velocity time graph.

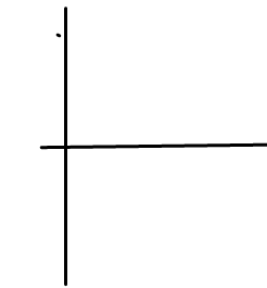
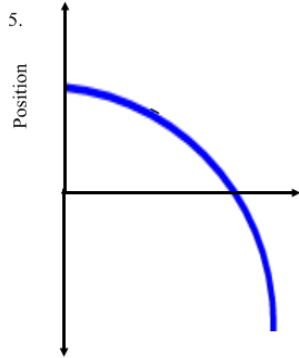
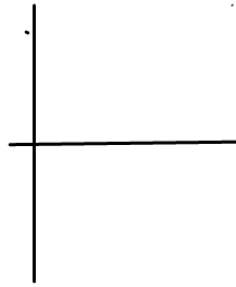
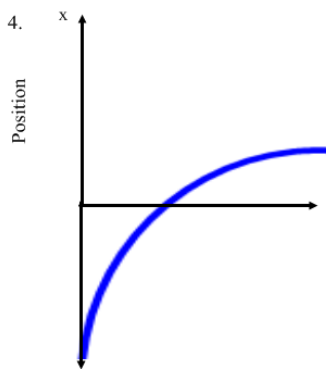
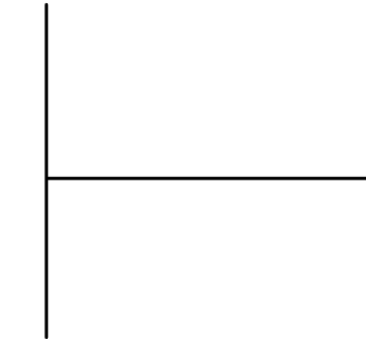
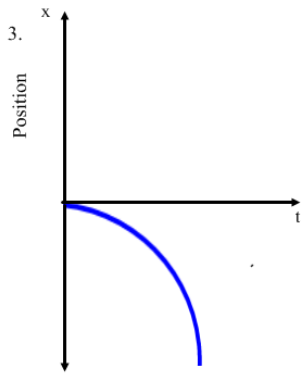


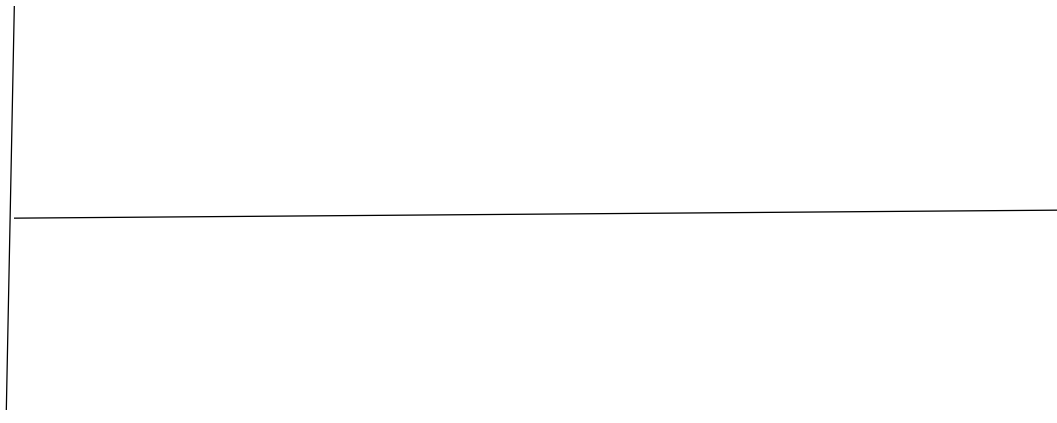
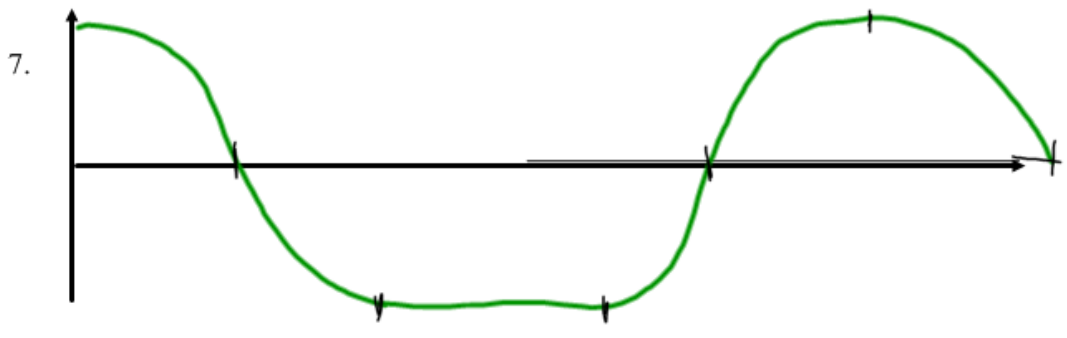
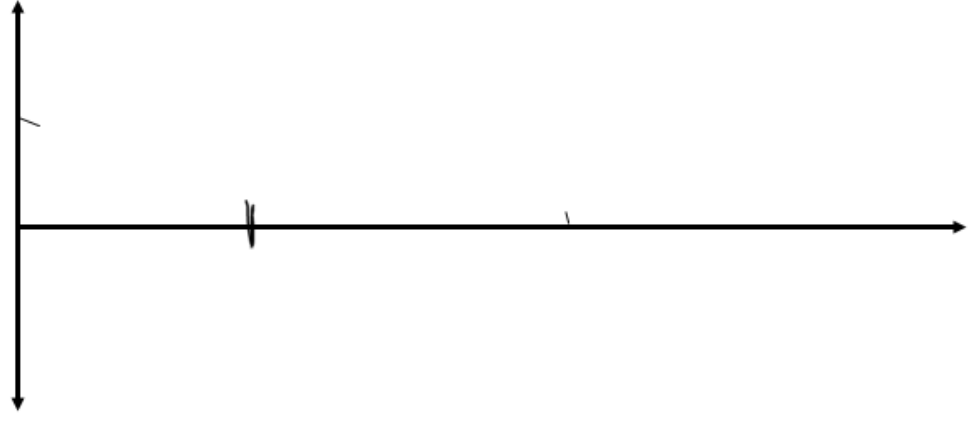
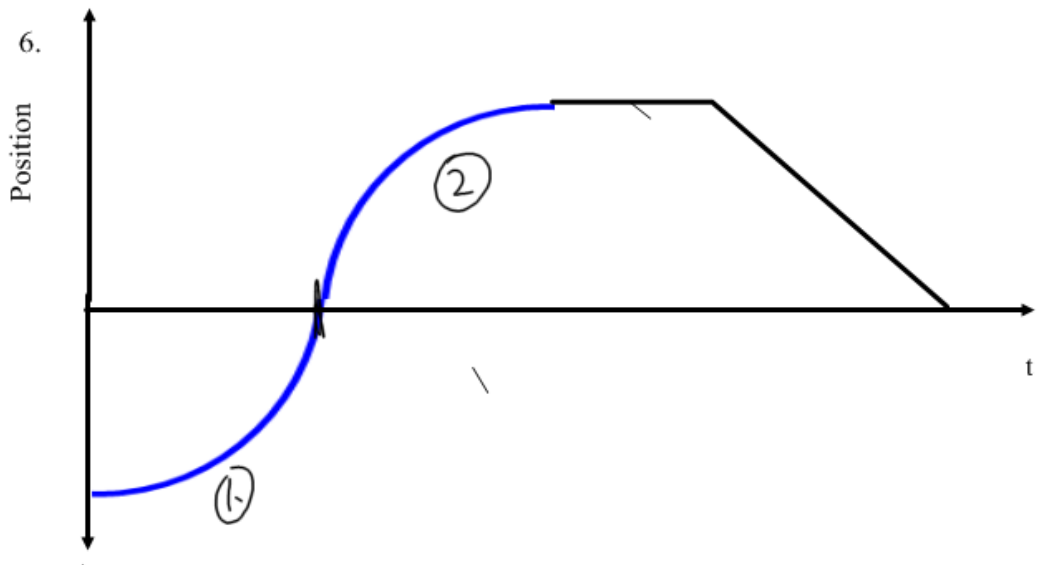
To create a V-t Graph from a P-t graph that isn't moving at a constant velocity

- Draw 3 tangent lines
- Determine whether the lines are getting steeper or flatter
(steeper means the velocity is increasing - flatter means decreasing)
- Determine if the lines are positive slope or negative slope
(Positive means the velocity is on the positive side - negative means the velocity is negative)
- If the position time graph starts on the negative side and moves to the positive side it simply means that the initial position was negative

Example 3.1



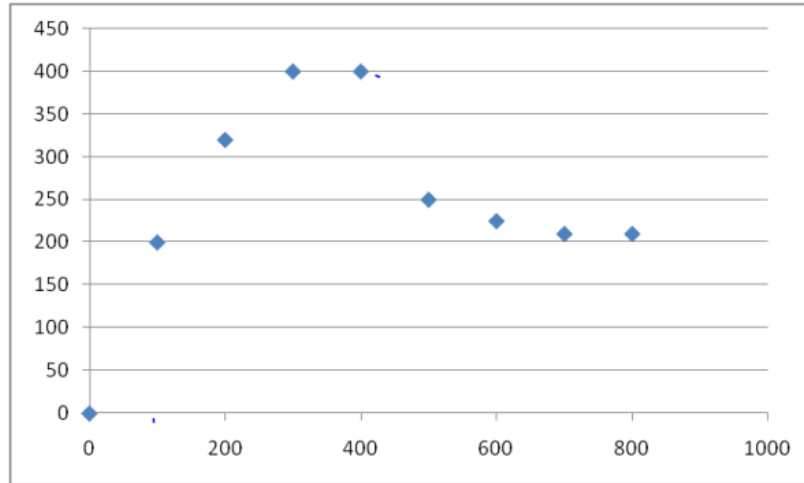




Example 3.2 – Didn't make the revision!!

Example 3.3 Use the following P-t graph to complete a V-t graph. For the curved sections you should complete all the constant velocities and zero velocities and then fill in the curved sections

$t(s)$	$x(m)$
0	0
100	200
200	320
300	400
400	400
500	250
600	225
700	210
800	210



Direction of Displacement, Velocity and Acceleration

Example 3.4

Hank heads south when he slams on his brakes. What is the direction of his velocity, displacement and acceleration, net force, momentum, distance

D

V

A

F_{net}

P

Distance



Motion Problems for Maximums/Minimums

We are going to need to use the quadratic formula to solve equations.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*We of course can replace x with t if we are looking for time

* You will remember that we need to have the equation in the form:

$$ax^2 + bx + c = 0$$

Example 3.5 Find the roots of the following equation:

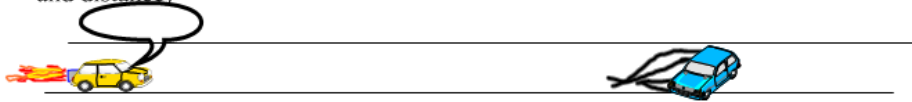
$$100 = 20t + \frac{1}{2}(5)t^2$$

For the following problems there are two types of scenarios.

Scenario 1: involves only one object and two different situations (constant velocity then slowing or accelerating then decelerating). In these situations we create two equations and substitute one into the other.

Scenario 2: involves 2 different objects. Both objects share at least two things in common (usually final position and final time). We arrange for the same variable in each and set the two equations equal to each other.

Example 3.6 Teresa is travelling on a secondary highway at 90.0km/h when she suddenly sees a truck (by truck I mean car) parked across the road 40.0m. It takes her 0.75 seconds to react and apply the brakes, which allow her to decelerate at -10.0m/s^2 . a) Will she hit the truck? b) What is the maximum velocity she could be travelling and not hit the truck? (Use the same reaction time and distance)



Example 3.7 Chipper is travelling at 50km/h and Davey, who is behind him, is travelling at 144km/h. After Davey passes him, Chipper starts accelerating at 4.5m/s^2 in an attempt to catch him. How long will it take Chipper to catch Davey if he waits 20 m to start accelerating.



Example 3.8 Peter and Brian are travelling at the same velocity beside each other when Brian starts to slow down at a constant rate. Create an equation to determine the distance between them when Brian is stopped.

Example 3.9 Abe is travelling at 20m/s when he decelerates for a certain distance of rough road. His deceleration is 1.5m/s^2 . He then accelerates again at 2.5m/s^2 until he reaches 132m and a velocity of 30m/s . How long was the rough section?



Example 3.10 Mariah and Dave are 60m apart in a demolition derby. Dave's car is a bit faster and can accelerate at 4m/s^2 . Mariah's car seems to turn better though and she is already moving at 4m/s when she accelerates at 2.5m/s^2 . If Dave started from rest determine how far they are from the center of the field when they collide. They collide head on although Mariah has already tried to run into Dave's driver's side door, which of course we all know is not really proper etiquette, nor is it legal.



Unit 4

Conservation of Momentum with Angles

Law of Conservation of Momentum - all of the momentum in a system prior to a collision or explosion is equal to all the momentum in the system after.

There are 3 basic situations involving momentum.

1. Elastic collisions - the objects involved in the collision are separate before the collision and are apart after the collision.
2. Inelastic collision - the objects are separate before the collision but are locked together after the collision
3. Explosions/Separations - the objects are together before the incident and separate after the incident

We will be looking at collisions that occur at angles. We must break the velocities into x and y components and find the resultant velocity at the end. It is a similar procedure to vector analysis.

Equations

1. Elastic collisions

$$m_a v_{ax} + m_b v_{bx} = m_a v_{ax}' + m_b v_{bx}'$$

$$m_a v_{ay} + m_b v_{by} = m_a v_{ay}' + m_b v_{by}'$$

* The velocities can be broken down in a table or we can replace each velocity with the appropriate equation using cos or sin.

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

2. Inelastic collisions

$$m_a v_{ax} + m_b v_{bx} = (m_a + m_b) v_x' \quad v' = \text{velocity of the objects once they are locked together}$$

$$m_a v_{ay} + m_b v_{by} = (m_a + m_b) v_y'$$

3. Explosions/Separations

$$(m_A + m_B) v_x = m_A v_{Ax}' + m_B v_{Bx}' \quad v = \text{velocity before separation (m/s)}$$

$$(m_A + m_B) v_y = m_A v_{Ay}' + m_B v_{By}'$$

Example 4.1 A 200g cue ball is shot at an angle of 90° at 4m/s. It collides with a 180g ball that is stationary. The cue ball rolls away at 3m/s at 60° . What is the velocity and direction of the other ball after the collision?

Given

$$m_c = 0.20\text{kg}$$

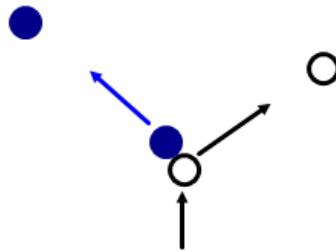
$$m_b = 0.18\text{kg}$$

$$v_c = 4\text{m/s at } 90^\circ$$

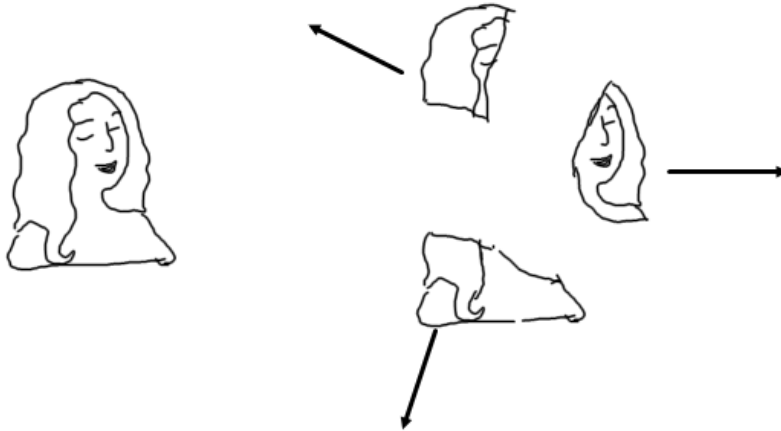
$$v_b = 0$$

$$v_c' = 3\text{m/s at } 60^\circ$$

$$v_b' = ?$$



Example 4.2 A large bust of Queen Elizabeth III is wired with small explosives. Upon detonation it exploded into 3 pieces. One piece (A) has a mass of 12kg and leaves at 25m/s at an angle of 0° . Piece B has a mass of 10kg and flies off at 40m/s at 135° . What is the velocity and direction of the third 20kg piece?



Example 4.3 The details are taken from the police report of a collision that happened in Yurtown, Canada. The time of the collision was May 8, 2008 at 3:45pm.

- First driver - Mr. Alvin Marcum was travelling in a car with a mass of 1000kg at a velocity of 50km/h and at an angle of 280° .
- As Alvin passes through the intersection he collides with one Mr. Clarence Jones, who is travelling in a 1200kg vehicle.
- Tests of the marks indicate that the two vehicles locked together and were travelling at 80km/h at a direction of 300° .
- Determine how fast Mr. Jones was travelling before the collision and the direction he was heading.

Conservation of Energy

Law of Conservation of Energy - energy cannot be created or destroyed only changed in form.

$$W = E_f - E_i$$

W = work (J) *can be positive or negative

E_f = the final energy that exists in a system (J)

E_i = initial energy that exists in a system (J)

Remember that energy can exist in many forms. The forms we will be using are:

1. Kinetic energy
2. Gravitational potential energy
3. Elastic potential energy

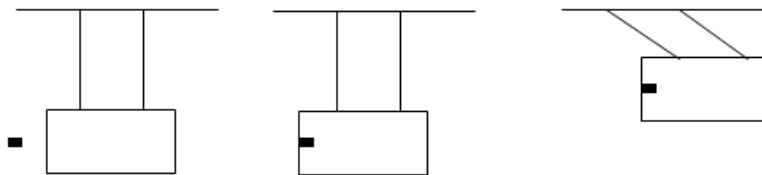
$$E_K = \frac{1}{2}mv^2$$

$$E_g = mgh$$

$$E_E = \frac{1}{2}kx^2$$

Example 4.4 A forensic expert is finding the initial velocity by firing a bullet into a ballistic pendulum. the bullet has a mass of 5.50g and the pendulum has a mass of 1.75kg. When the bullet hits the pendulum it swings up to a maximum height of 12.5cm before swinging back. Determine

- a) the velocity of the bullet and pendulum upon contact (**energy**)
- b) the velocity of the bullet before it made contact with the pendulum (**momentum**)



We need to solve the second part (energy) first to find the velocity of the bullet and pendulum. We can then work backwards and find the velocity of the bullet before it hit the pendulum.

Example 4.5 - Collision Reconstruction: Details

- Car A (Lexus) collided with Car B (Toyota)
- Car B was parked at time of collision
- Vehicles lock together
- Marks indicate vehicles travelled 12m after collision
- Coefficient of friction is 0.80
- Mass of Lexus is 1300kg, mass of Toyota is 2000kg
- velocity of Lexus before collision must be determined

of



Assumptions

Momentum

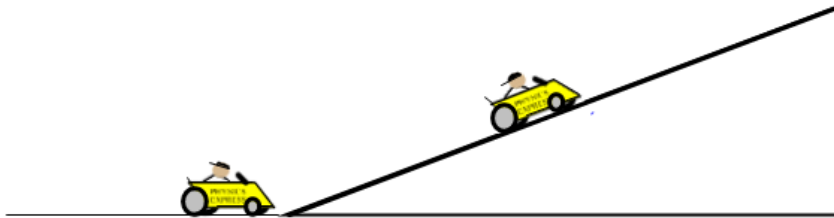
$$v_T = 0\text{m/s}$$

Assumptions

Energy

vehicles come to a complete stop at the end
accident happens on level ground

Example 4.6 You and your friends are experimenting. They push you in your coaster car towards a ramp that is 15m long and has an angle of 30° . How far up the ramp will you make it if the force of friction is 509.22N and your velocity at the bottom is 8m/s. The total mass is 150kg. (*Remember : F_{II} is not acting against you in this method)



Heat Energy

During situations where energy is converted from one form to another heat is lost as a by product.

$$Q = mc\Delta T$$

or

$$Q = mc(T - T_0)$$

Q = heat energy (J)

m = mass of material (kg)

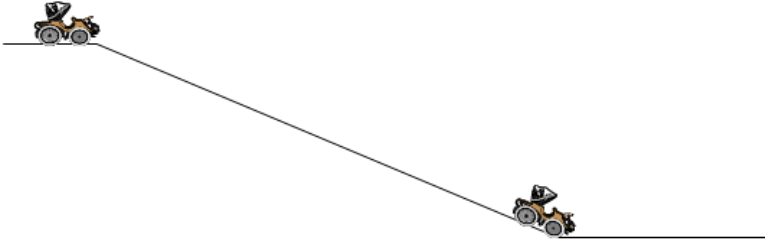
c = specific heat capacity of material ($\text{J}/\text{kg}^{\circ}\text{C}$)

ΔT = change in temperature ($^{\circ}\text{C}$)

Example 4.7 A 4kg block of iron is heated from 20°C to 80°C . Determine the amount of heat energy absorbed by the iron.

Remember that the energy can not be created or destroyed only changed in form. Thus far we have always determined the work done to slow down or stop an object. Where does the energy go? Usually it is lost as heat or sound. If an object is stopping the parts that stop the vehicle can heat therefore the work done to stop the object is lost energy which another part of the object gains as heat energy. In the next example we will assume all the energy lost in stopping the car is gained equally by all four brakes.

Example 4.8 A compact car of early 20th century, European origin and a mass of 900kg is travelling at 72km/h when it reaches the top of a hill. The top of the hill is 10m higher than the bottom of the hill. If Hans, who is driving the car, keeps the car at a constant velocity for the whole length of the hill determine a) the work done by the brakes to keep the car from accelerating b) the heat gained by the brakes if they are made of iron and they each have a mass of 20kg.



Projectile Motion

Projectiles in basic terms are anything that flies through the air and are affected by only gravity. We will neglect air resistance as it is its own science.

The path of a projectile will look like a parabola if you graphed height vs. time.

The following equations will be used to determine time, height, displacement, velocity, etc

Vertical Displacement

$$y = y_0 + v_{y0}(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

y = height (m)

y_0 = initial height (m)

v_{y0} = initial velocity in the y-direction (m/s)

a = acceleration (in this case gravity) (-9.8m/s^2)

t = time (sec)

t_0 = initial time (sec)

Horizontal Displacement (a.k.a Range)

$$x = x_0 + \bar{v}_x(t - t_0)$$

x = position (m)

x_0 = initial position (m)

\bar{v}_x = velocity in the x-direction (m)
(this stays constant through the path of the projectile)

Vertical Velocity with Time

$$v_y = v_{y0} + g(t - t_0)$$

v_y = final velocity in the y-direction (m/s)

g = acceleration of gravity (-9.8m/s^2)

✓ *This equation can be used to determine the time to the maximum height.

Vertical Velocity with Displacement

$$v_y^2 = v_{y0}^2 + 2g(y - y_0)$$

✓ * This equation can be used to find the maximum height.

Determining Time in the Air

- use quadratic formula with vertical displacement equation
- use t_{up} and t_{down} equations

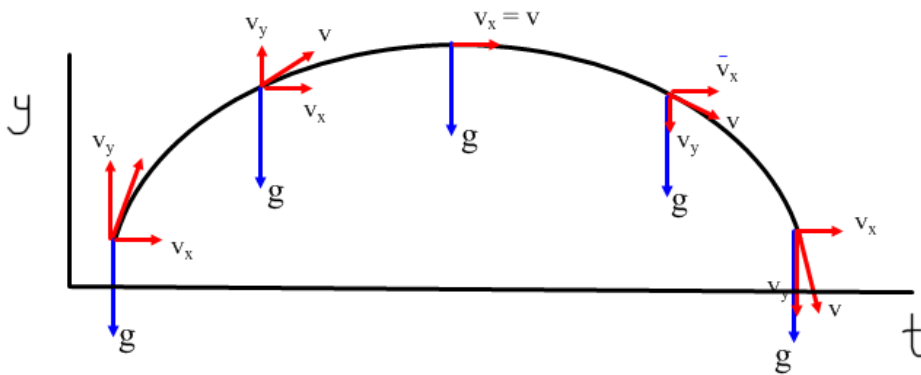
Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Remember

$$v_x = v \cos\theta \qquad v_y = v \sin\theta$$

Velocities and Accelerations in a Path of a Projectile



There are 3 basic scenarios for projectile motion:

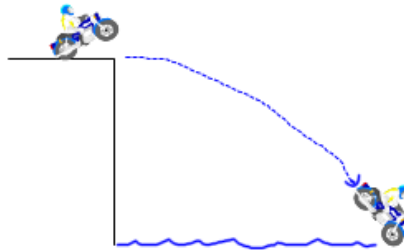
1. Object is launched horizontally ($v_{y0} = 0\text{m/s}$)
2. Object starts and lands at the same height ($y = y_0$, you don't need the quadratic formula)
3. Object starts and land at different heights (need quadratic formula or t_{up} and t_{down})

1. Objects Launched Horizontally

The y-component of the initial velocity is 0. We can use the previous formula to find time in the air, range, etc.

Example 4.9 Woodrow drives his motorbike off a cliff for a movie stunt. If his velocity was 20m/s determine:

- a) His time in the air if the cliff is 40m above the water.
- b) How far from the cliff he lands.



Example 4.10 A bird is flying with a fish in its claws. The bird is flying at a height of 30m at a velocity of 8m/s. The bird drops the fish in an attempt to land it in a nest that is 20m high and 12m ahead of it in the horizontal direction. Is it successful?



nest dia
= 5dm

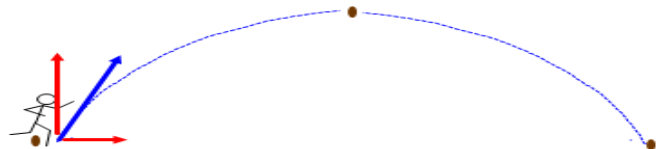
2. Objects that start and land at same height

**Remember - y and y_0 are equal*

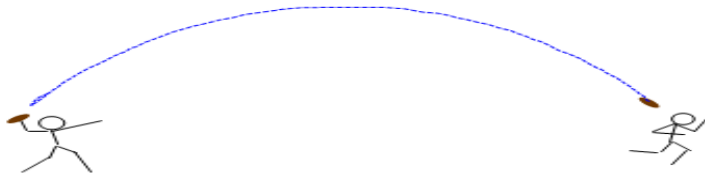
- break the velocity into x and y components

Example 4.11 Valerie kicks a football with an initial velocity of 20m/s at an angle of 35° . Determine the following values:

- time to maximum height
- maximum height
- time from maximum height to the ground
- time in the air
- range



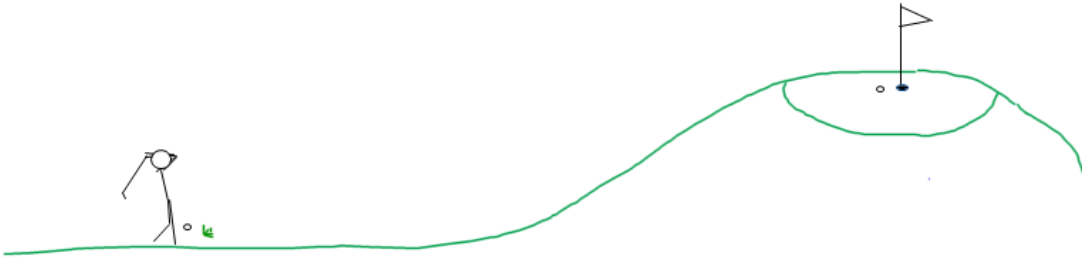
Example 4.12 A football quarterback throws the ball with an initial velocity of 20m/s at an angle of 40° . The ball leaves at a height of 1.6m and is caught at a height of 1.6m . a) How long is the ball in the air? b) What is the maximum height? c) How far down the field does it land? d) If the receiver starts out 30m downfield how fast does he have to run to catch the ball assuming he leaves at the same instant the ball is thrown.



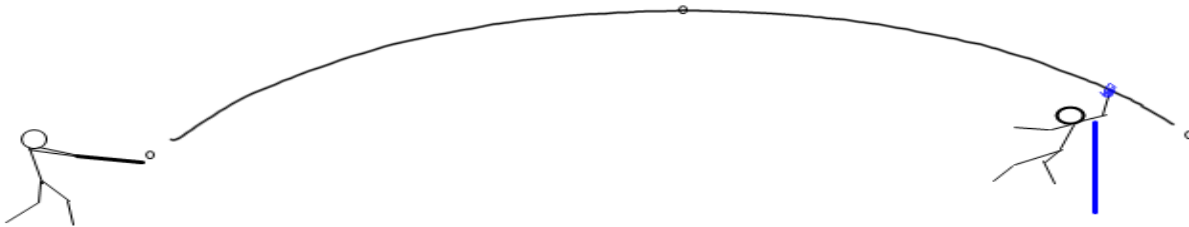
3. Projectiles Launched at an Angle and Don't Start and Land at the Same Height
(We will need to use the quadratic equation or t_{up} and t_{down})

Example 4.13 Shane hits a golf ball off a tee with an initial velocity of 55m/s at an angle of 35° . The ball lands on the green just 3m short of a hole in one. If the green is 6.3m higher than the tee find:

- time in the air
- distance from the tee to the hole
- velocity of the ball just before it hits the ground



Example 4.14 Kirby is playing baseball with his friend Hank. Hank pitches the ball and Kirby hits it 0.50 seconds after it was pitched. The ball left his bat at a velocity of 120km/h at an angle of 40° from a height of 1.2m. Will the ball clear a 8ft tall fence that is 110m away?



Example 4.15 Daniel shoots his potato gun at an angle of 40° from a height of 1m. The potato leaves the gun with a velocity of 20m/s. He is trying to hit a target on the ground that is 40m away. However, he hits a bird that is flying by. The bird is 12m away horizontally when it is hit. a) How high was the bird flying? b) Would he have hit the target if it was a 1m wide hoola hoop?



Symmetrical Trajectories

There are special equations for range and maximum height that work when the projectile **starts and lands at the same height. ($y = y_0$)**

Range

$$x = x_0 + v_x(t - t_0)$$

$$t_{\text{up}} = \frac{-v_{y0}}{g}$$

$$\bar{v}_x = v_0 \cos \theta$$

$$v_{y0} = v_0 \sin \theta$$

Combine these

$$x = x_0 + v_x(t - t_0)$$

$$x = v_0 \cos \theta (t)$$

$$x = v_0 \cos \theta \left(\frac{2(-v_{y0})}{g} \right)$$

$$x = -v_0 \cos \theta \left(\frac{2v_0 \sin \theta}{g} \right)$$

$$x = -v_0^2 \left(\frac{2 \sin \theta \cos \theta}{g} \right)$$

$$t = t_{\text{total}} = t_{\text{up}} + t_{\text{down}}$$

$$t = t_{\text{up}} + t_{\text{up}}$$

$$t = 2t_{\text{up}}$$

$$t = 2 \left(\frac{-v_{y0}}{g} \right)$$

Since the halves of the trajectory are equal

$$t_{\text{up}} = t_{\text{down}}$$

Trig Identity

$$\sin 2\theta = \boxed{}$$

$$R = \frac{-v_0^2 \sin 2\theta}{g}$$

Height (Maximum)

$$v_y^2 = v_{y0}^2 + 2g(y - y_0)$$

$$v_{y0} = v_0 \sin \theta$$

Combine the above equations (we will assume y_0 equals 0 but if not you will have to add it after)

$$v_y^2 = v_{y0}^2 + 2g(y - y_0)$$

$$0 = (v_0 \sin \theta)^2 + 2g(y_{\text{max}})$$

$$y_{\text{max}} = \frac{-v_0^2 \sin^2 \theta}{2g}$$

$$H = \frac{-v_0^2 \sin^2 \theta}{2g} + y_0$$

Time in the Air

$$T = \frac{-2v_0 \sin \theta}{g}$$

Example 4.16 A wooden dowel is launched from a projectile launcher at an angle of 30° and lands 6m away. Determine the initial velocity of the launcher. You can assume for simplicity sake that the dowel will land at the same height as it was launched at.

Example 4.17 Using the launcher above determine which of the following angles would give the greatest range a) 28° b) 45° or c) 62°

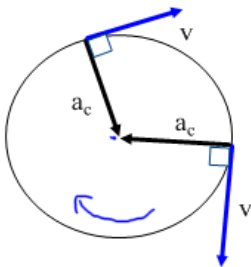
Example 4.18 Using the same launcher again determine the angle you need to set the launcher at to hit a target at 6.5m. How long would it be in the air ?

Uniform Centripetal Motion

- occurs when an object moves in a circle and its speed is constant

Centripetal Acceleration

- acceleration felt by an object as it moves in a uniform circular motion
- the acceleration is always directed toward the center of the circle along the radius
- the velocity is always tangent to the curve (it points in the direction the object would go if it went off the path of the circle)
- the velocity and direction is always tangent, it doesn't curve when it is released from the circle



$$a_c = \frac{v^2}{r}$$

a_c = centripetal acceleration (m/s²)
 v = velocity of object (m/s)
 r = radius of circle or curve (m)

In order for an object to continue in a circle some force must exist that is directed towards the center. This force is an overall force, basically a net force for circles and is called ***centripetal force***.

Centripetal force is not a regular force like frictional force or force of gravity or tension in a rope. It is an overall heading that includes one or more of these forces or whatever keeps the object moving in a circle. It is basically the net force in a circle.

For example: 1. gravity is the force that keeps the moon orbiting the earth. 2. Friction is the force that keeps a car moving in a circle on a curved road. 3. Tension keeps a rock spinning in a circle around your head.

Centripetal Force

$$F_c = ma_c$$

OR

$$F_c = \frac{mv^2}{r}$$

F_c = centripetal force (N)

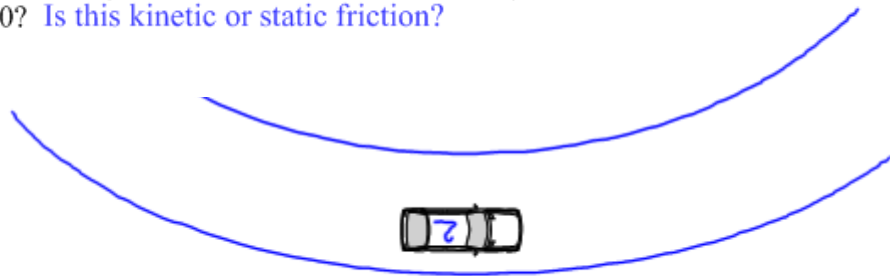
m = mass (kg)

a_c = centripetal acceleration (m/s^2)

Example 4.19 Georgetta is swinging a bag around her head in a circle by its strap. The length of the strap is 50cm and the velocity is 3m/s.

- What is the acceleration?
- What is the centripetal force if the mass is 5kg?
- What is the force of tension in the strap?

Example 4.20 A 1500kg car is driving around a curve on a level road. The curvature of the road (radius) is 50m. How fast can the car go around the curve without losing control if the coefficient of friction is 0.70? *Is this kinetic or static friction?*



Determining velocity in a circle

We know that the distance covered by a rotating object is equal to the circumference and the time it takes is the period we can combine the following formulas to create an equation for velocity in a circle.

$$c = 2\pi r$$

$$T = t/N$$

$$v = d/t$$

$$v = \frac{2\pi r}{T}$$

OR

$$v = \frac{2N\pi r}{t}$$

v = velocity (m/s)

r = radius (m)

T = period/time to complete one revolution(s)

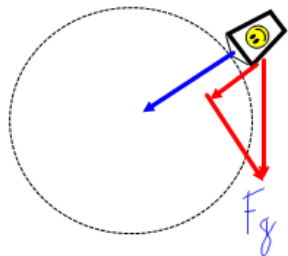
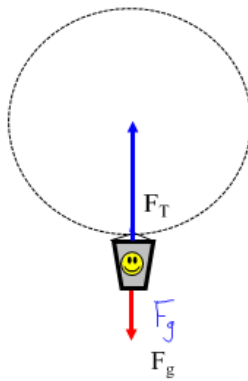
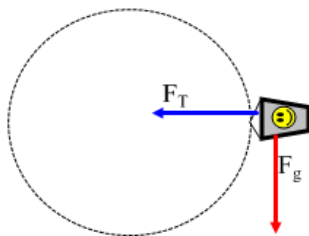
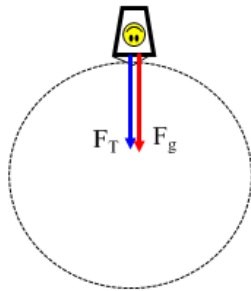
t = time (s)

N = number of cycles

Example 4.21 Norma is spinning around in a bucket that rotates in a circle around a fix point and is attached by a cable. Find the force of tension in the cable if the cable is 4m long and the bucket does 5 cycles in 20 seconds. The total mass of Norma and the bucket is 60kg.



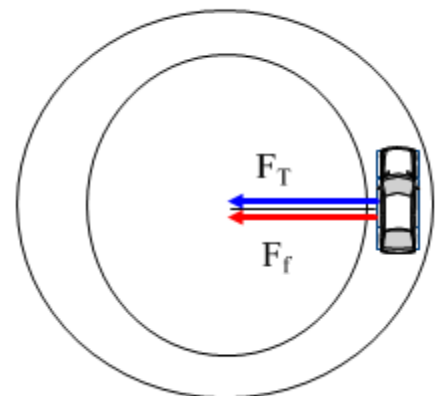
Situations with More than One Force Acting in a Circle



Example 4,22 Nona is swinging a pail of water in a vertical circle. The pail has a mass of 4kg and the length of her arm and the handle is 0.9m. Determine the following:

- a) Minimum speed she can swing the pail and not have water fall out. (We determine the minimum speed at the top since tension is the smallest at this spot, force of tension will be zero at the minimum speed)
- b) The force of tension on her shoulder at the side and bottom of the swing.

Example 4,23 A test track is designed to measure forces on a body as it rotates. The apparatus is designed such that the vehicle drives around a small track in a circle. To increase the force it is also attached to a pivot by a cable. The vehicle has a mass of 500kg and the maximum tension of the cable is 30000N. Determine the maximum velocity of the vehicle if the coefficient of friction is 0.25. Determine the centripetal force experienced by the driver. The radius of the track is 40m.



Centripetal Force And Banked Curves

We have talked about the forces that keep an object rotating such as gravity, friction and tension. However, objects can go around a turn in a road without friction provided that the curve is banked.

Through the derivation of equations the equation becomes

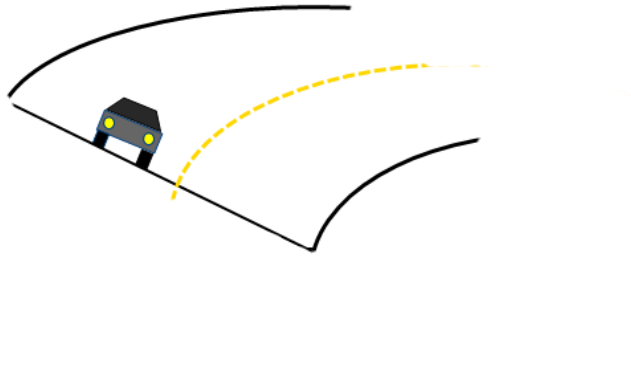
$$\tan \theta = \frac{v^2}{rg}$$

r = radius of curve (m)

v = velocity (m/s)

θ = angle of the banking (degrees)

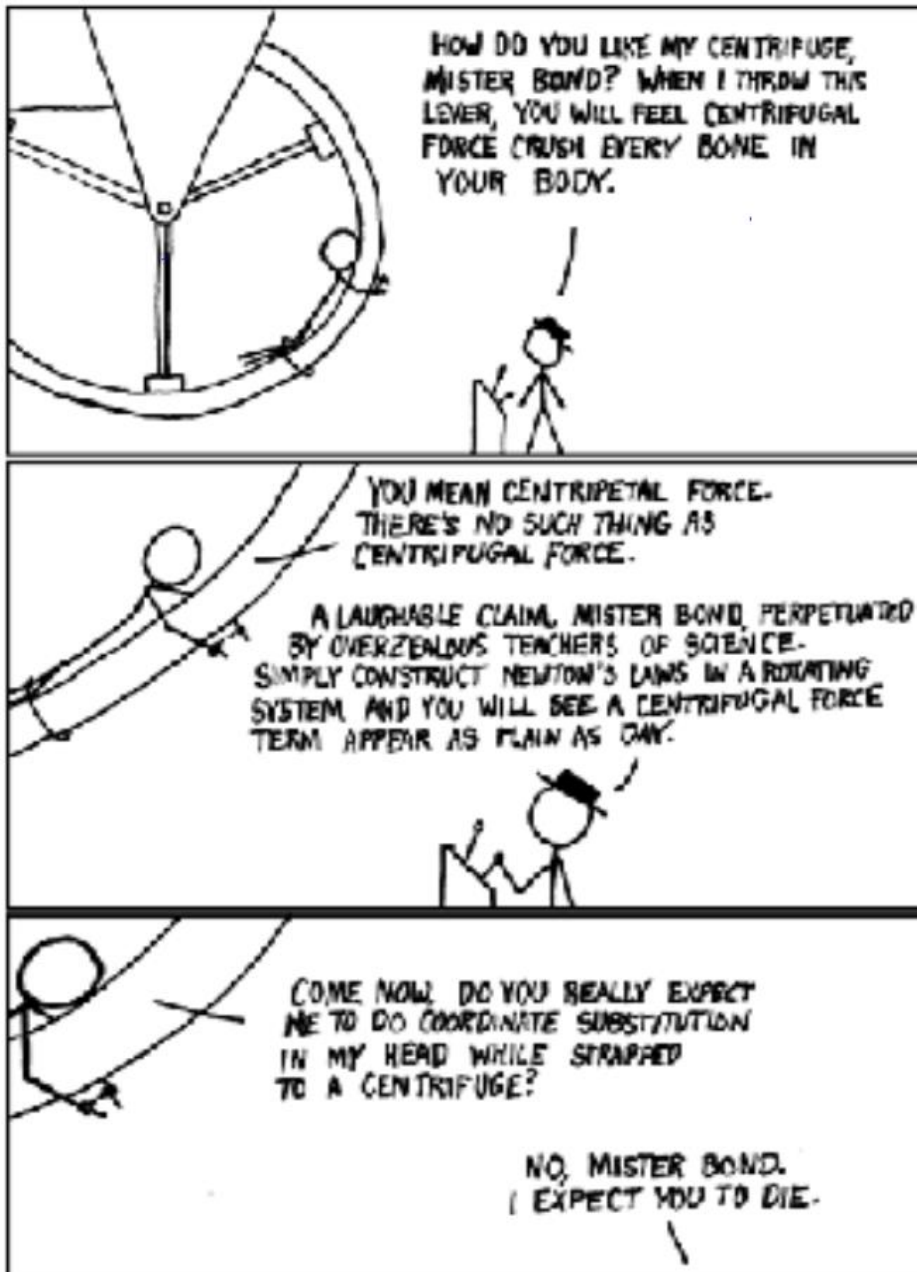
Example 4.24 An engineer is designing an off-ramp that is curved. He has designed it so that the radius of the curve is 100m. What would the banking need to be to allow a car to take the ramp at 72km/h without the aid of friction.



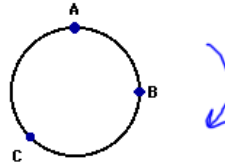
Centripetal vs. Centrifugal Force

Centripetal force is a force that keeps you moving in a circle. It is a real force.

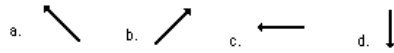
Centrifugal force is a fictitious force. It is the name used to represent the force you feel that you think is trying to throw you out of a car when you are going in a circle. In actuality the inside of the car keeps you in the car and puts a force inward on you. If centrifugal force existed you would shoot straight out of the car in the direction of the radius. You would actually go in a direction tangent to the circle because of inertia.



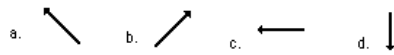
For questions #1-#5: An object is moving in a clockwise direction around a circle at constant speed. Use your understanding of the concepts of velocity, acceleration and force to answer the next five questions. Use the diagram shown at the right. Answers are provided in the pop-up menu below.



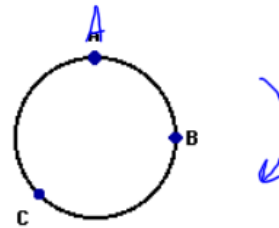
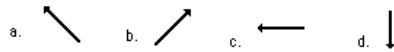
1. Which vector below represents the direction of the force vector when the object is located at point A on the circle?



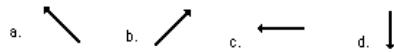
2. Which vector below represents the direction of the force vector when the object is located at point C on the circle?



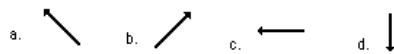
3. Which vector below represents the direction of the velocity vector when the object is located at point B on the circle?



4. Which vector below represents the direction of the velocity vector when the object is located at point C on the circle?



5. Which vector below represents the direction of the acceleration vector when the object is located at point B on the circle?



6. Rex Things and Doris Locked are out on a date. Rex makes a rapid right-hand turn. Doris begins sliding across the vinyl seat (which Rex had waxed and polished beforehand) and collides with Rex. To break the awkwardness of the situation, Rex and Doris begin discussing the physics of the motion which was just experienced. Rex suggests that objects which move in a circle experience an outward force. Thus, as the turn was made, Doris experienced an outward force which pushed her towards Rex. Doris disagrees, arguing that objects which move in a circle experience an inward force. In this case, according to Doris, Rex traveled in a circle due to the force of his door pushing him inward. Doris did not travel in a circle since there was no force pushing her inward; she merely continued in a straight line until she collided with Rex. Who is correct? Argue one of these two positions.



Rotational Angle and Angular Velocity

We have been talking about velocity in this unit. The velocity is the linear velocity, the velocity the object would go in a straight line. We also want to look at angular velocity.

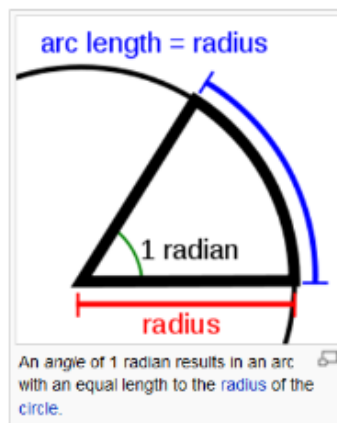
Angular Velocity - rate of change of an angle, can be in rads/sec or revolutions/ sec
- basically the same as frequency of a rotating object

$$\omega = \frac{\Delta\theta}{t}$$

ω = angular velocity (rads/sec)
 $\Delta\theta$ = change in angle (radians)
 t = time (sec)

Lets look at some of the following equations to relate to centripetal acceleration.

First, what is a radian?



We want to discuss rotational angle at this time so that we can use it later.

Rotational angle is $\Delta\theta$ and it is the ratio of the arc length to the radius of curvature. If the arc length and the radius are the same then the ratio would be 1 radian, the same as the definition above.

$\Delta\theta = \frac{\Delta s}{r}$	$\Delta\theta = \text{rotational angle (radians)}$
	$\Delta s = \text{arc length or distance travelled in a circular path (m)}$
	$\text{(if greater than 1 you can multiply by the number of rotations)}$
	$r = \text{radius of curvature of circular path (m)}$

If we use the equation for linear velocity in a way that uses distance travelled in a given amount of time we would get:

$v = \frac{\Delta s}{t}$	$v = \text{velocity (m/s)}$
	$\Delta s = \text{arc length or distance travelled in a circular path (m)}$
	$t = \text{time (sec)}$

Rearranging the equation above for Δs we get

$\Delta s = r\Delta\theta$ which we can sub into the velocity equation to get

$v = \frac{r\Delta\theta}{t}$ This will give us:

$v = r\omega$	$v = \text{velocity (m/s)}$
	$r = \text{radius of curvature (m)}$
	$\omega = \text{angular velocity (rad/s)}$

Example 4.25 Old vinyl records used to operate at constant velocity when they were played. A full length album was called a 12" record and it turned at 33 1/2 times per minute. a) Determine the revolutions per second. (frequency) b) Determine the linear velocity at a point on the outer edge of the record. c) Determine the angular velocity.

Centripetal Acceleration With Angular Velocity

If we combine the following formulae

$$v = r\omega \quad \text{AND} \quad a_c = \frac{v^2}{r}$$

Replacing for v gives:

$$a_c = r\omega^2$$

Example 4.26 Calculate the centripetal acceleration of a point 7.50cm from the axis of an ultra-centrifuge spinning at 75,000 rpm. Relate the acceleration to gs.

Given

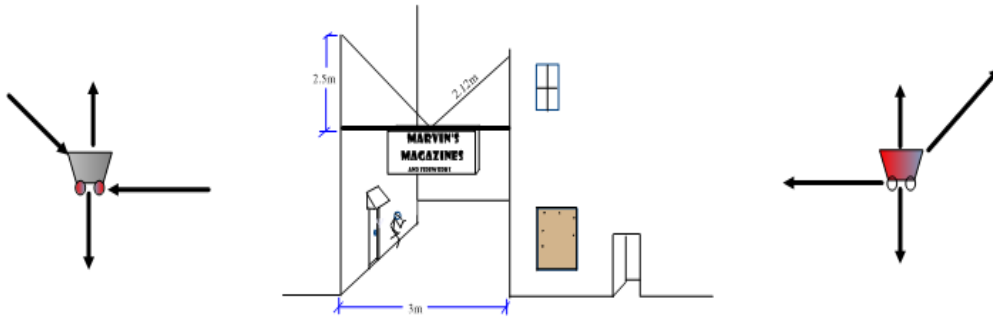
$$r = 0.075\text{m}$$

$$\omega = 75000\text{rpm} = \quad \text{rad/s}$$

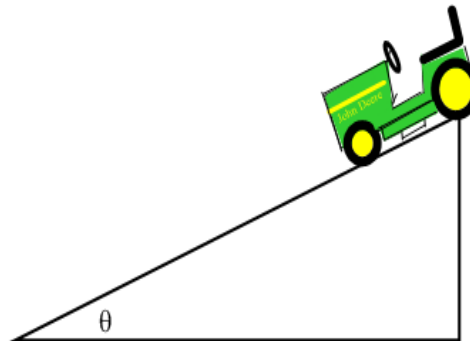
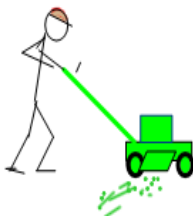
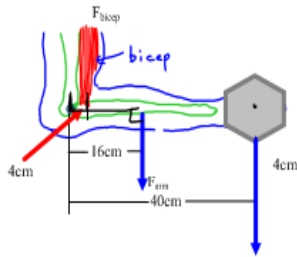
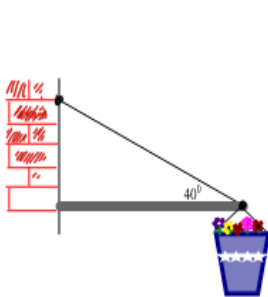
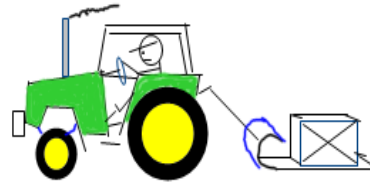
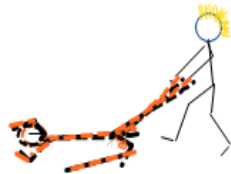
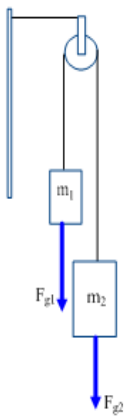
$$a = ?$$

Convert the angular velocity from rpm to rad/s

$$\frac{75000\text{rev}}{1\text{min}} \left| \frac{2\pi \text{ rad}}{1 \text{ rev}} \right| \left| \frac{1\text{min}}{60\text{sec}} \right| = 7854 \text{ rad/s} \leftarrow \text{This is } \omega.$$



Forces



Dynamics Extension - Welcome to Forces with Angles

Key Ingredients

1. Applied Force - a push or pull
- F_A

2. Frictional Force - acts in a direction opposite to the direction of intended motion
- F_f

$$F_f = \mu F_N$$

3. Force of gravity - self explanatory - Weight
- F_g

$$F_g = mg$$

4. Normal force - force a surface puts on an object.

★ - *the normal force is the equal to the force of gravity **plus/minus** the vertical force of the applied force

- F_N

- more explanation to come

5. Net Force - sum of all the forces in a plane (i.e all the forces in the x-direction or all the forces in the y-plane or all the forces in a parallel plane)

- most important concept of forces

- F_{net}

$$F_{net} = \text{sum of all the forces in a plane}$$

* there is no standard, one size fits all, formula

* you must draw a FBD and create an equation

6. Free Body Diagram - FBD

- dot diagram that shows all of the forces acting on an object

- used to create the F_{net} equation

- you must draw them and fill in the missing values

Newton's 3 Laws of Motion

First Law - a.k.a. Inertia

- a body at rest will stay at rest until an external force acts upon it

OR

- a body in motion will stay in constant motion until an external force acts upon it

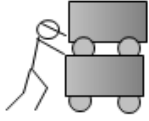
Second Law

- the net force on an object is equal to its mass multiplied by acceleration

Third Law - Law of Retaliation

- For every force there is an equal and opposite force

Example 5.1 A small cart has a mass of 50kg and is pushed with a force of 120N. The coefficient of friction 0.204. Determine the acceleration.



Example 5.2 James is driving down the road when a large crate he was hauling in the back of his truck falls out. The coefficient of the friction between the crate and the road is 0.14 and the crate has a mass of 30kg. If the truck was travelling at 60km/h when the crate falls out, how far will it(THE CRATE) go before it stops?

Motion with Forces at Angles

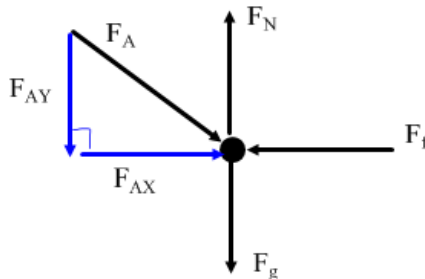
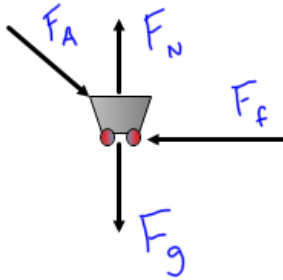
Key Points

1. When we have a force at an angle we must break it into x and y components.
Remember x is the horizontal component and y is the vertical component.
2. The normal force is equal to the force of gravity PLUS or MINUS the y-component of the applied force.

Forces Applied at Angles can be broken into 2 main types:

1. Force is being pushed forward and downward
2. Force is being pulled forward and upward

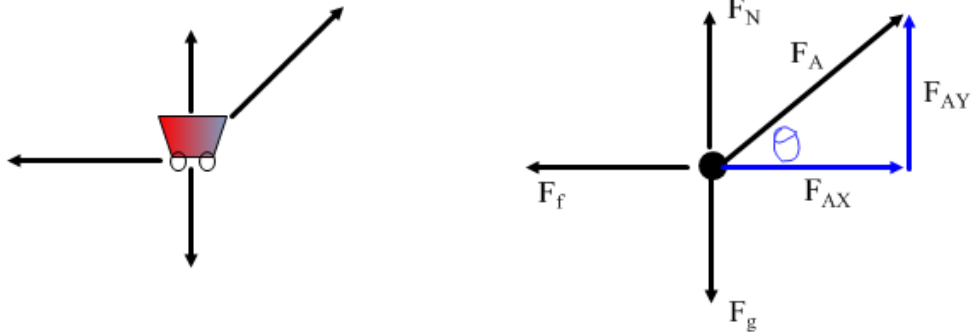
1. Force is Forward and Down



Create an F_{net} equation for the x-direction and the y-direction

Since we can assume that the object will not move upward or downward we can rearrange the $F_{\text{net}Y}$ equation to get F_N knowing that $F_{\text{net}Y}$ will be zero

2. Force pulling up and forward



Create an F_{net} equation for the x-direction and the y-direction

Since we can assume that the object will not move upward or downward we can again rearrange the $F_{\text{net}Y}$ equation for F_N knowing that $F_{\text{net}Y}$ will be zero.

To summarize the normal force

1. When the force is applied downward
 - add the y-component to the force of gravity
2. When the force is applied upward
 - subtract the y-component from the force of gravity

Example 5.3 Luke is pushing a lawnmower with a force of 80N. The handle makes an angle of 54° with the horizontal. If the mower has a mass of 20kg and the coefficient of friction is 0.15, determine the F_{AX} , F_{AY} , equation for normal force and the acceleration.

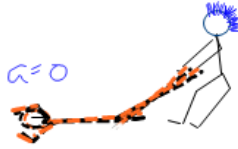


Example 5.4 Harold is pulling a sled with an applied force of 550N. The force is applied at an angle of 60° and the mass of the sled is 60kg. Determine the acceleration. Coefficient of friction is 0.48.





Example 5.5 Calvin is dragging his friend Hobbes down a hallway at a constant velocity. Hobbes' mass is 80kg and the coefficient of friction is 0.48. If he applies a force at an angle of 60° , determine the force he applies. How much force does he have to apply if the angle is 45° .



Example 5.6 Cory is pushing an ice cream trike with a downward force at a constant velocity of 2.45543 m/s. (Dat dare chain done broked an' falled off) The trike has a mass of 100kg. If the coefficient of friction is 0.25 and the force of friction is 300N, find the applied force and the angle.

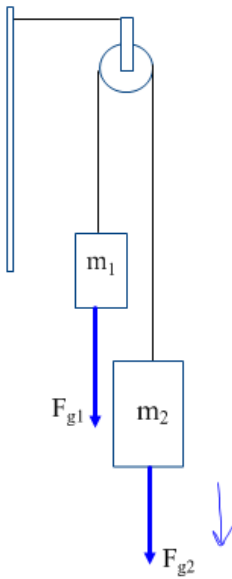


Analyzing Motion in Connected Objects

The *Atwood Machine* was created to help understand the forces in objects that are connected. It is a simple pulley and masses arrangement.

For the purposes of analysis we need to redraw the apparatus as a horizontal system and apply all the forces. Since they are connected we need to use the total mass to find the acceleration.

Atwood Machine



Redraw it horizontally and create a net force equation for the whole apparatus.



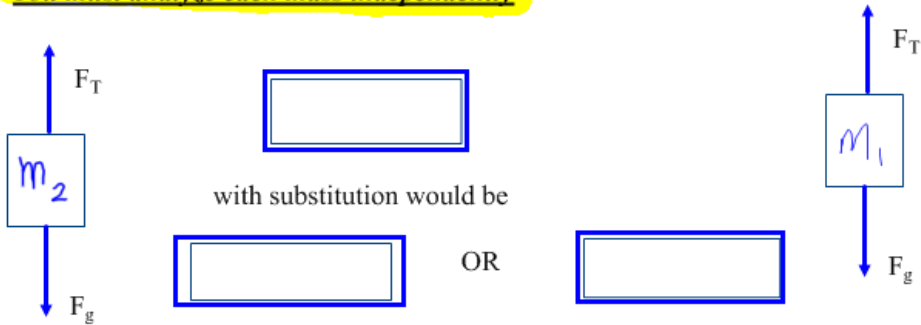
with substitution would be



* The acceleration will tell you which mass will move upward and which will move downward.

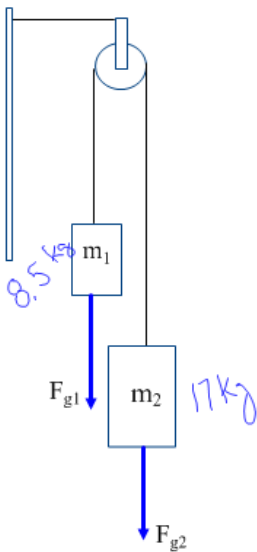
Finding Tension in the Cable/String

You must analyze each mass independently

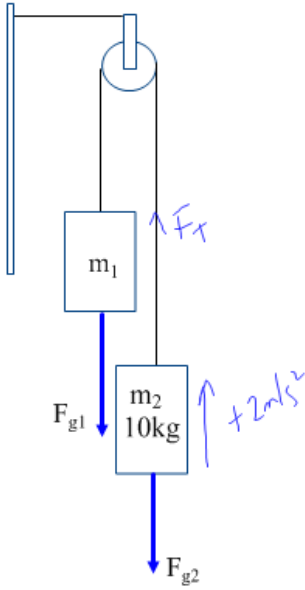


- * The F_T is the same for both equations
- * The F_{net} is based on only one mass and is NOT the same as the net force in the first part
- * The acceleration is (+) if the mass is going upward and (-) if going down.

Example 5.7 An Atwood Machine is set up as shown below. Determine the acceleration of the masses and the force of tension in each side of the cable given that mass 1 is 8.5kg and mass 2 is 17kg.



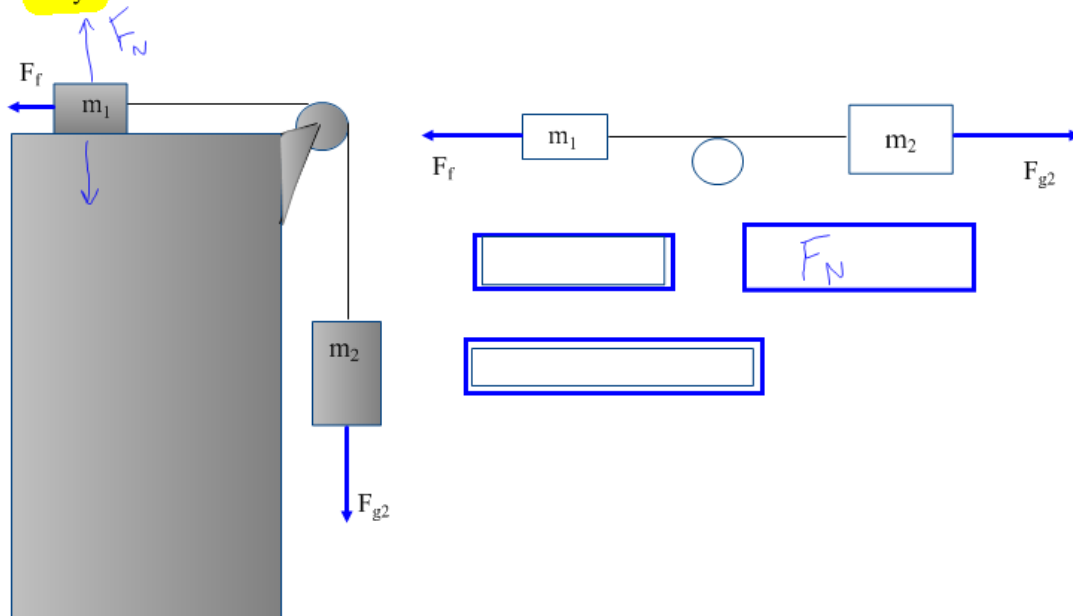
Example 5.8 An atwood machine is set up as shown below. Determine the mass of the unknown mass if its acceleration is 2.0m/s^2 downward.



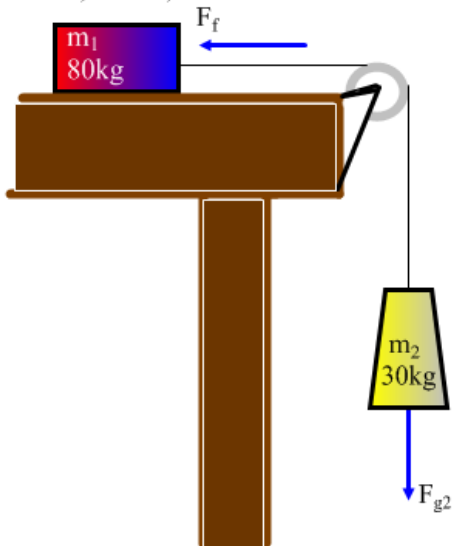
Variation of The Atwood Machine (Modified Atwood Machine)

It is an Atwood Machine with one object on a flat surface and the other object is hanging
It is still based on the same principles with the exceptions:

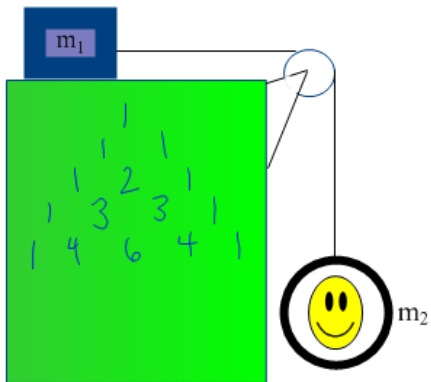
1. The force of friction is the force acting against the force of gravity.
2. The mass on the table can slide in the direction of the pulley but it will not slide the other way.



Example 5.9 A large red and blue block with a mass of 80kg is sitting on a dark brown oak table. It is attached to a yellow and grey block that is hanging on a string over a pulley as shown below. Determine the acceleration of the blocks if the coefficient of friction is a) 0.30 b) 0.60



Example 5.10 You are trying to determine what mass block would need to be used for the modified Atwood machine below. The block on the table is attached to a hanging 30kg ball. When the 30kg ball is released the acceleration of the unit is 1.2m/s^2 . The coefficient of friction is 0.404.



Inclined Planes

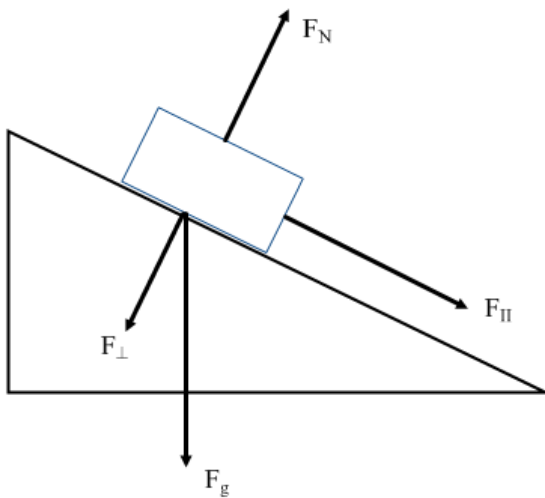
- These are simply ramps or slopes
- We do not use F_x and F_y for inclined planes because you are not working on the x and y axis
(*Note: some university profs will call the surface the x-axis and the perpendicular is used as the y-axis)
- Because we are working at an angle we are going to be using the terms parallel and perpendicular to describe the direction of the forces and motion

Parallel Force - F_{\parallel}

- force that acts parallel to the surface of the ramp, incline, etc.
- it is the component of the object's weight that allows it to slide down the ramp

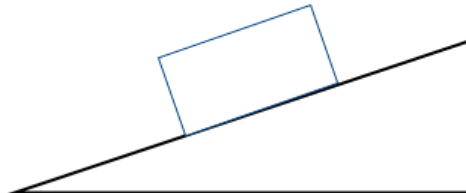
Perpendicular Force - F_{\perp}

- force that acts perpendicular to the surface of the ramp
- it is the component of the object's mass that acts against the surface and affects friction
- it is equal to the normal force (assuming any applied forces are directed along the surface of the ramp)



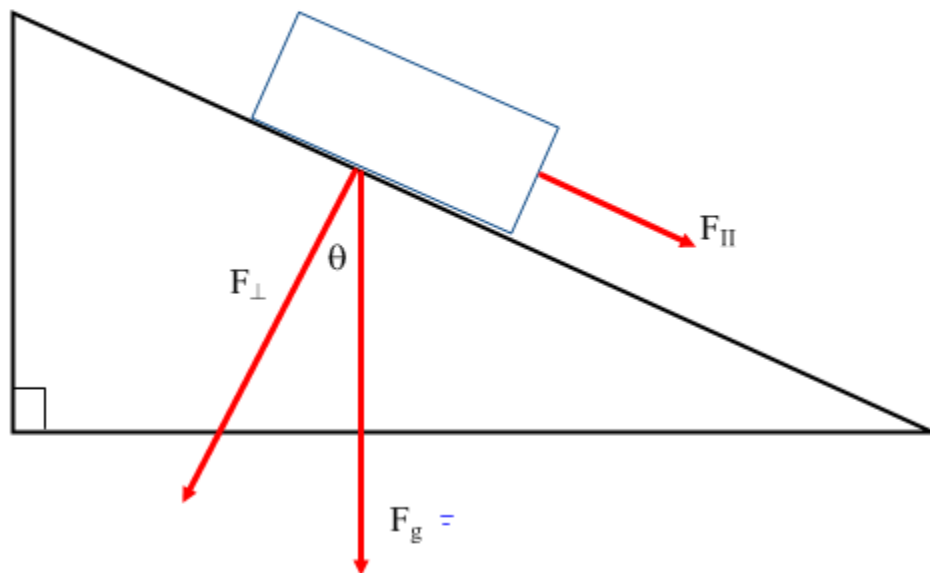
* F_f depends on the direction of the block.

***We will use to the right as positive and to the left as negative regardless of the slope of the ramp.**



Example 5.11 Determine the force required to push a blue box full of used coat buttons on a ramp at a constant velocity if the F_{\parallel} is 300N and the F_f is 450N when the direction is a) upward b) downward.

Determining Parallel and Perpendicular Forces



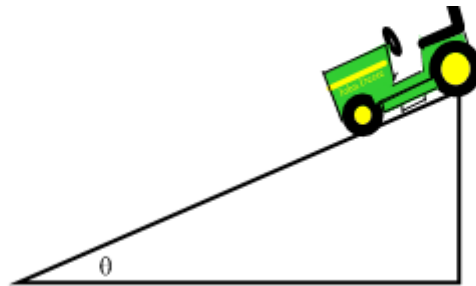
Example 5.12 A 40kg box sits on a ramp that has an angle of 30° . Determine the acceleration of the box if the coefficient of friction is 0.25.

Example 5.13 A box with a mass of 15kg sits on a 5m long ramp at the bottom. If the angle of the ramp is 40° and the coefficient of friction is 0.25 determine the following:

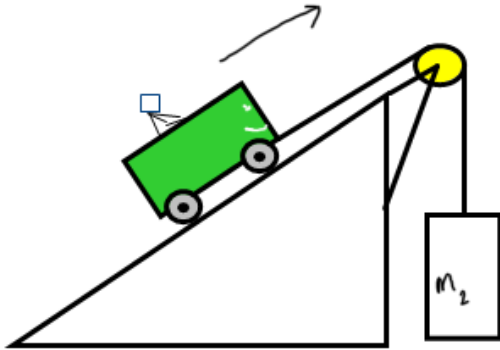
- Force required to accelerate the box up the ramp with an acceleration of 1m/s^2
- Velocity at the top of the ramp. (your positions will be based upon using the surface of your ramp not the horizontal or vertical distances.)

Example 5.14 A 100kg box is placed on a ramp. If the coefficient of friction is 0.40 determine the maximum angle that will still keep the box stationary.

Example 5.15 A 300kg ride-on John Deere lawnmower is placed at the top of a ramp with the park brake set as shown below. The park brake is released and the lawnmower starts to roll down the ramp. The ramp is 4m long and it is 2m high at the starting end. If the velocity of lawnmower is 2.5m/s at the bottom calculate the coefficient of friction.



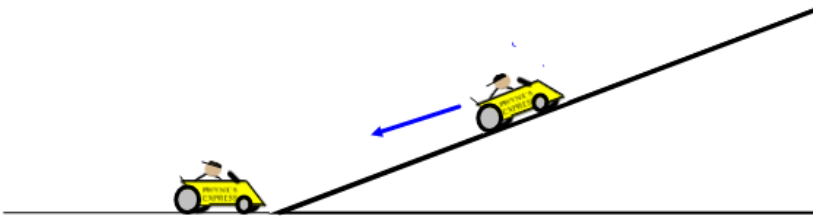
Example 5.16 A hanging mass is being used to pull a large cart up a ramp. The hanging mass has a mass of 200kg and the cart has a mass of 250kg. The coefficient of friction is 0.10 and the angle of the ramp is 60° . Determine the acceleration of the cart.



Conservation of Energy and Inclined Planes

We are going to combine the principles of energy and ramps. For these questions we still use F_f but F_{II} is covered by the energy part. We need F_{\perp} to determine the force of friction and consequently the work. (We have already done this question using Conservation of Energy)

Example 4.6 You and your friends are experimenting. They push you in your coaster car towards a ramp that is 15m long and has an angle of 30° . How far up the ramp will you make it if the coefficient of friction is 0.4 and your velocity at the bottom is 8m/s. The total mass is 150kg. (*Remember : F_{II} is not acting against you during Conservation of Energy but it does if using F_{net})



Efficiency of Ramps

$$\text{Eff} = \frac{W_o}{W_I} \times 100\%$$

W_o = work output (J)

W_I = work input (J)

Work output is the amount of work done to an object in an ideal situation. In the case of a ramp, the work output would be the energy it has at the top of the ramp - E_g .

Work input would be the force applied multiplied by the displacement in the parallel direction. If you don't have the applied force you would need to calculate it just as you would in the previous questions.

Example: You are pushing a 100kg crate up a ramp at a constant velocity with a force of 915N. The ramp is set at an angle of 30° and is 5m long.

a) Determine the efficiency of the ramp.

b) Determine the efficiency of the ramp if it is set at 45° and the applied force is 1040N.

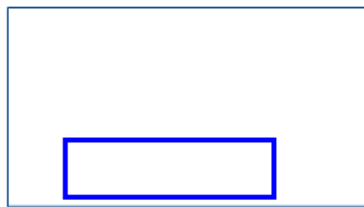
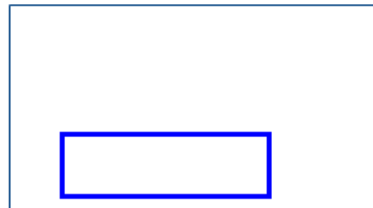
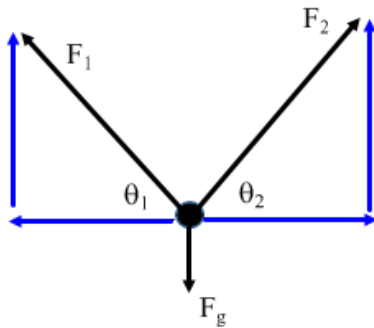
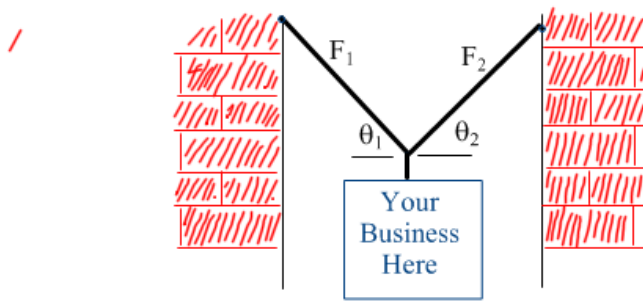
Static Equilibrium - Forces in Equilibrium

This unit is about the analysis of forces in which there is no movement (static) and all forces are balanced (in equilibrium). Some people will call this unit Statics.

Key Points

- $F_{\text{net } x} = 0$ (which means all the forces to the right = all the forces to the left)
- $F_{\text{net } y} = 0$ (all forces acting upward = all forces downward)
- If you use any substitutions you must state the original equation
- We must draw a FBD and break all forces into x-components and y-components

Most examples are objects supported from cables such as hanging signs and at higher levels it is used for analysis of bridges.



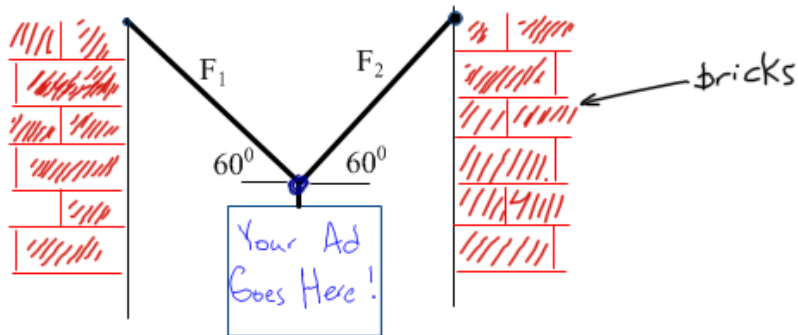
If the angles are equal and we know that $F_{x1} = F_{x2}$ then



Steps to solving static equilibrium questions

1. Draw FBD and label forces
2. Break all forces at angles into x and y-components
3. Solve for unknowns using the equations above
4. You may need to use substitution for F_y to help solve.

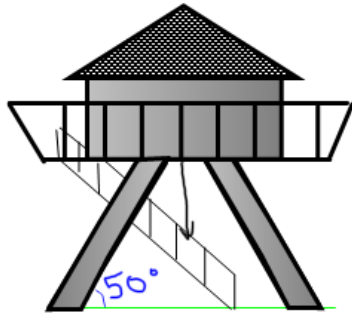
Example 5.17 Determine the force in each cable below required to hold the 50kg sign.



The force in each cable is 282.9N.

***The important point to solving these problems is stating the reasons why you are doing certain changes or stating certain answers. If $F_{y1} = F_{y2}$ you need to state why. You can't just say they are equal or substitute without an equation from which the substitution is made.**

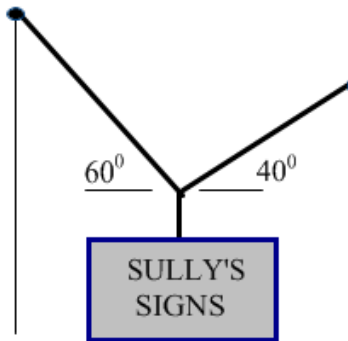
Example 5.18 Determine the forces in each of the beams that support the 10000kg building below. The angle between the beams and the ground is 50°



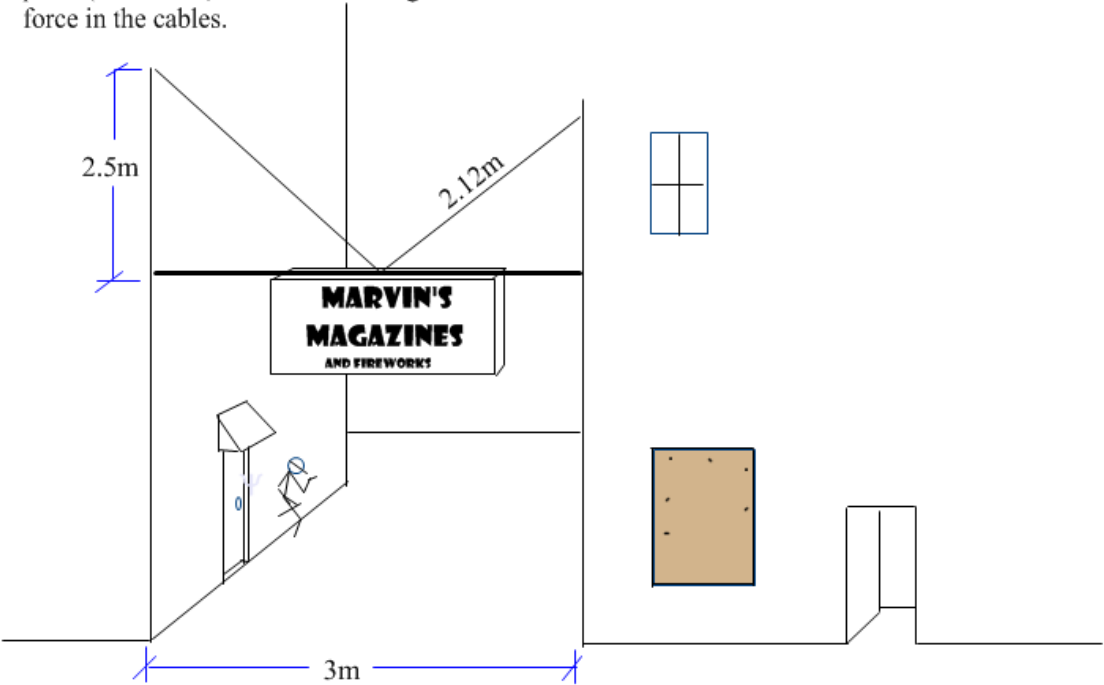
Unequal Angles

- We are going to need another way to find F_{y1} and F_{y2} as they aren't equal.
- Remember to state any equations you use for substitution

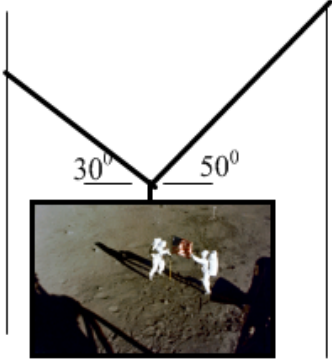
Example 5.19 Determine the force in each cable below. The mass of the sign is 80kg.



Example 5.20 A 30kg sign is to be hung in an alley directly between two buildings. The buildings are 3m apart and height from the top of the sign to the first attachment point (on the left) is 2.5m. The length of the second cable is 2.12m. Determine the force in the cables.



Example 5.21 George is trying to determine how heavy his sign can be (in pounds) if he sets it up as shown in the diagram below. The force in cable 2 is 800N. The force in cable 1 is unknown.



Torque

Torque is a force that causes rotational motion.

Torque is equal to the applied force multiplied by the perpendicular distance from the point of rotation.

τ - tau

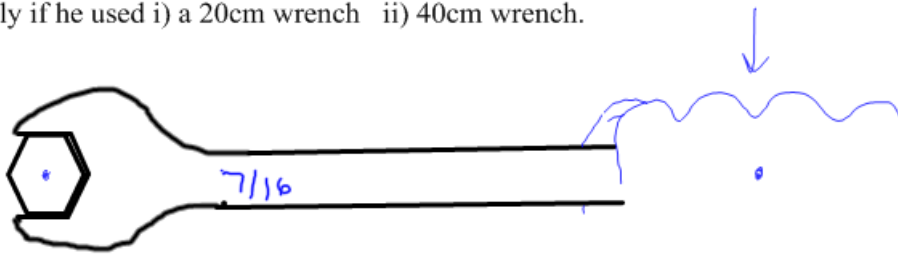
$$\tau = Fr_{\perp}$$

τ = torque (Nm) (16 .ft)

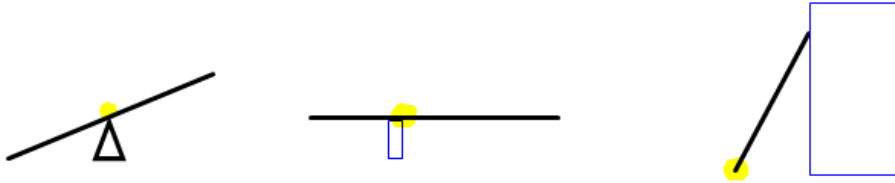
F = applied force (N)

r = perpendicular distance (m)
(sometimes called the lever arm)

Example 5.22 Bryant is torquing the bolts on his car and they are to be set at 100 lb ft of torque. a) Convert to Nm b) Determine how much force he would need to apply if he used i) a 20cm wrench ii) 40cm wrench.



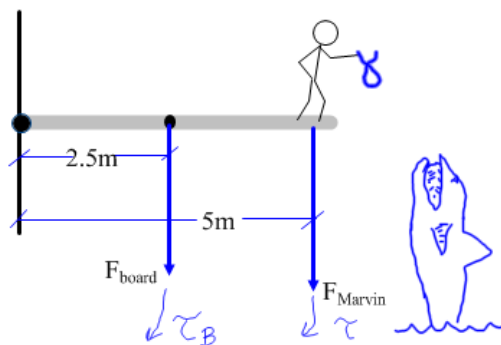
Pivot Point (aka the point of rotation) - the point about which an object will rotate. It can be the middle of a see saw, the support point under board or the base of a ladder.



Example 5.23 Marvin is standing at the end of a diving board. His mass is 80kg and the board has a mass of 5kg. Determine the torque in each situation:

- The board is 5m long and the pivot point is at the wall
- The board is 6m long and the pivot is 2m from the other end.

Note: we will assume that the mass of a board, pole, etc., is concentrated at the center, regardless of the location of the pivot point



Static Torque

In many situations with rotation forces we can find missing or unknown force by finding the net torque. For stationary objects the net forces and the net torque must equal zero.

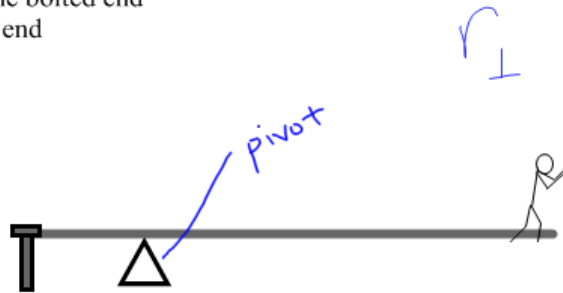
For example: if a see saw is to be balanced and is not moving the torque on both sides must be the same. A smaller person can balance a big person by sitting further from the pivot point (if possible of course)

We will use clockwise rotation as positive and counterclockwise as negative.

Example 5.24 Carl is standing at the end of a plank that is 5m long. The plank has a 5kg and Carl has a mass of 60kg. At the other end the board is held down with a large bolt.

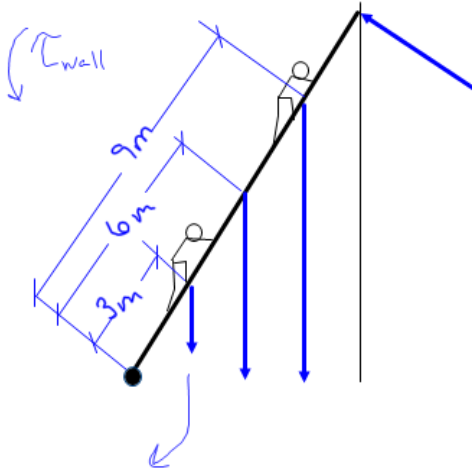
Determine the force in the bolt when the pivot point is:

- 1m from the bolted end
- 2.5m from the bolted end
- at the bolted end

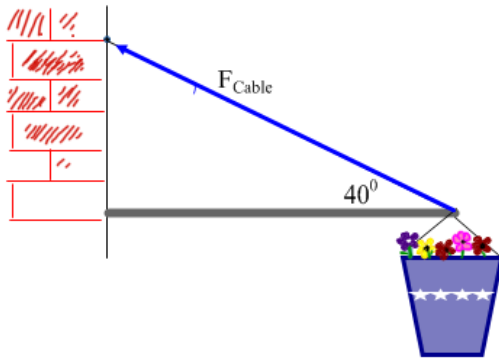


Sometimes the object in the question is at an angle or one of the forces is at an angle. In order to calculate the torque we need to determine the perpendicular distance using trig ratios.

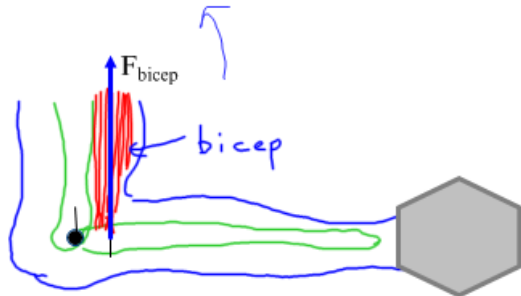
Example 5.25 Ellen and Mark are both standing on a ladder that is leaning against a wall. The ladder makes an angle of 60° . The ladder itself is 10kg and is 12m long. Ellen is $1/4$ of the way up the ladder and has a mass of 50kg. Mark is $3/4$ of the way up the ladder and has a mass of 80kg. Determine the force the building puts on the ladder to keep it from rotating. Use the bottom of the ladder as the pivot point.



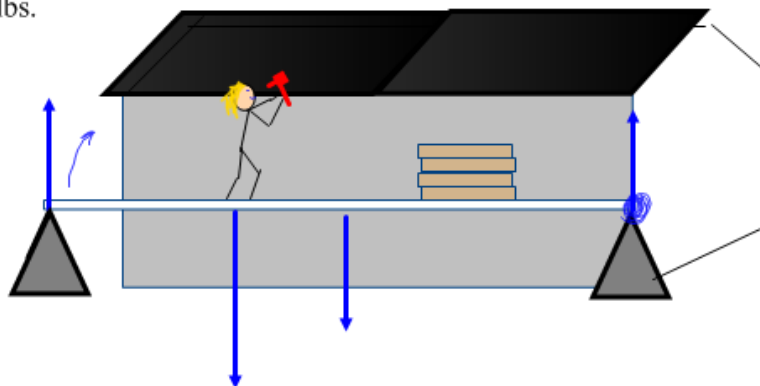
- * Example 5.26 In the diagram below the flower pot is hung at the end of a pole that is connected to the side of a house. The pole has a mass of 6kg and is 2.5m long. The pole is supported by a single cable that makes an angle of 40° with the pole. The maximum force (of tension) that the cable can support is 800N. Determine the maximum mass of the flower pot. Determine the force that the building puts on the pole



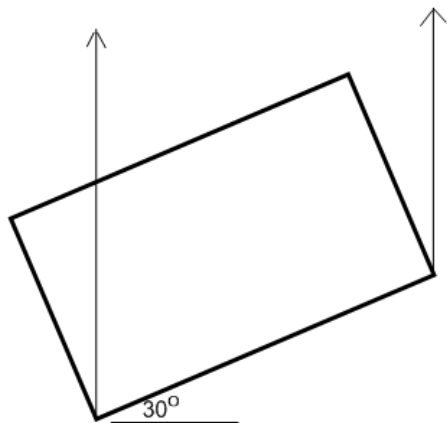
Example 5.27 Calculate the force in the biceps required to hold a 400N weight horizontally (* the mass of the lower part of the arm is 2.5kg)



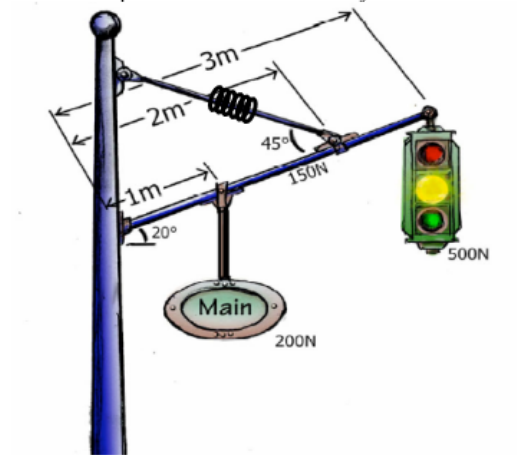
Example 5.28 Carla has decided she is going to put a new roof on her shed. She has devised a set up that should allow her to put the bundles on a plank and allow her to stand on it at the same time. Carla weighs 130lb and the shingles weigh 80lbs. Using the diagram below determine the force that must be supported by each end. The plank weighs 10lbs.



Example 5.29 Two cables are used to lift a 50lb. sheet of plywood. The plywood is a standard 4' x 8' size but is being lifted in a such a way that it is at a 30° angle with the horizontal. Determine the force in each cable. The weight is still centered in the middle of the board.

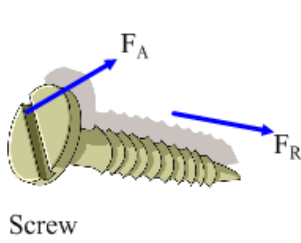


Example 5.30 The street light support system that is shown below has had some issues with breakage in the past so it has been proposed that a spring be placed in the upper support cable. The spring is expected to stretch 10cm past its unloaded length of 15cm. Specify the necessary spring constant required in order for this system to work.

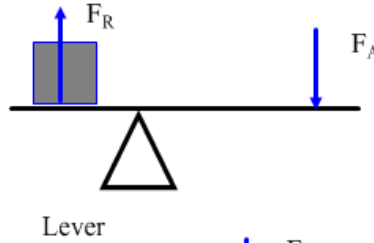


Simple Machines

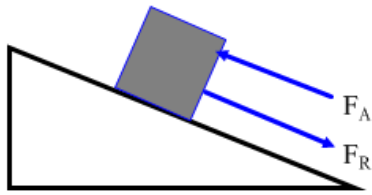
Simple machines are devices - levers, ramps, gears, pulleys, wedges, wheel and axle and screws - used to apply forces and torques. They may make it easier to apply a force because of the direction or they may increase the force applied on an object substantially.



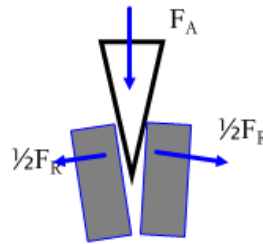
Screw



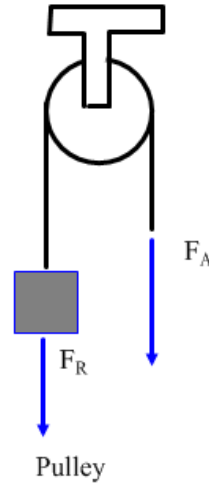
Lever



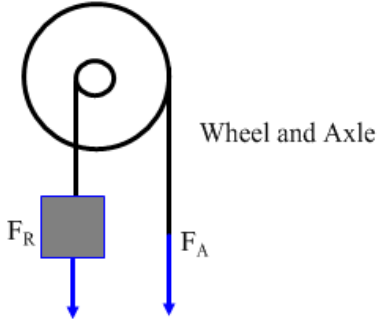
Ramp



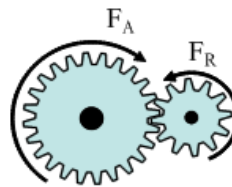
Wedge



Pulley



Wheel and Axle



Gears